On the discourse between advances in mechanical algebra and natural philosophy, as pertaining to the furtherment of enlightenment in the Ages of the Earth

Duke Andrew Ridgwell


EZIONE STRATIGRAFICO
OMAGNETICA
GOLA DEL BOTTACC

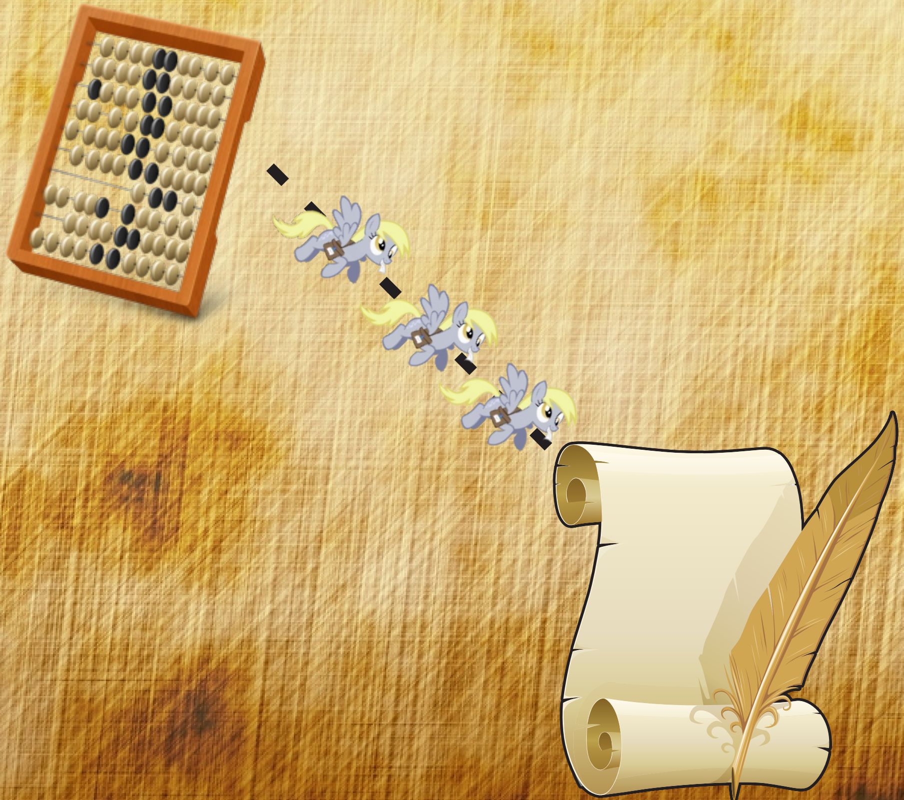
A Device for Mechanical Algebra

CORO-UNO

('Valdes' chipset)

A Device for Mechanical Algebra

CORO-UNO
(’Valdes’ chipset)



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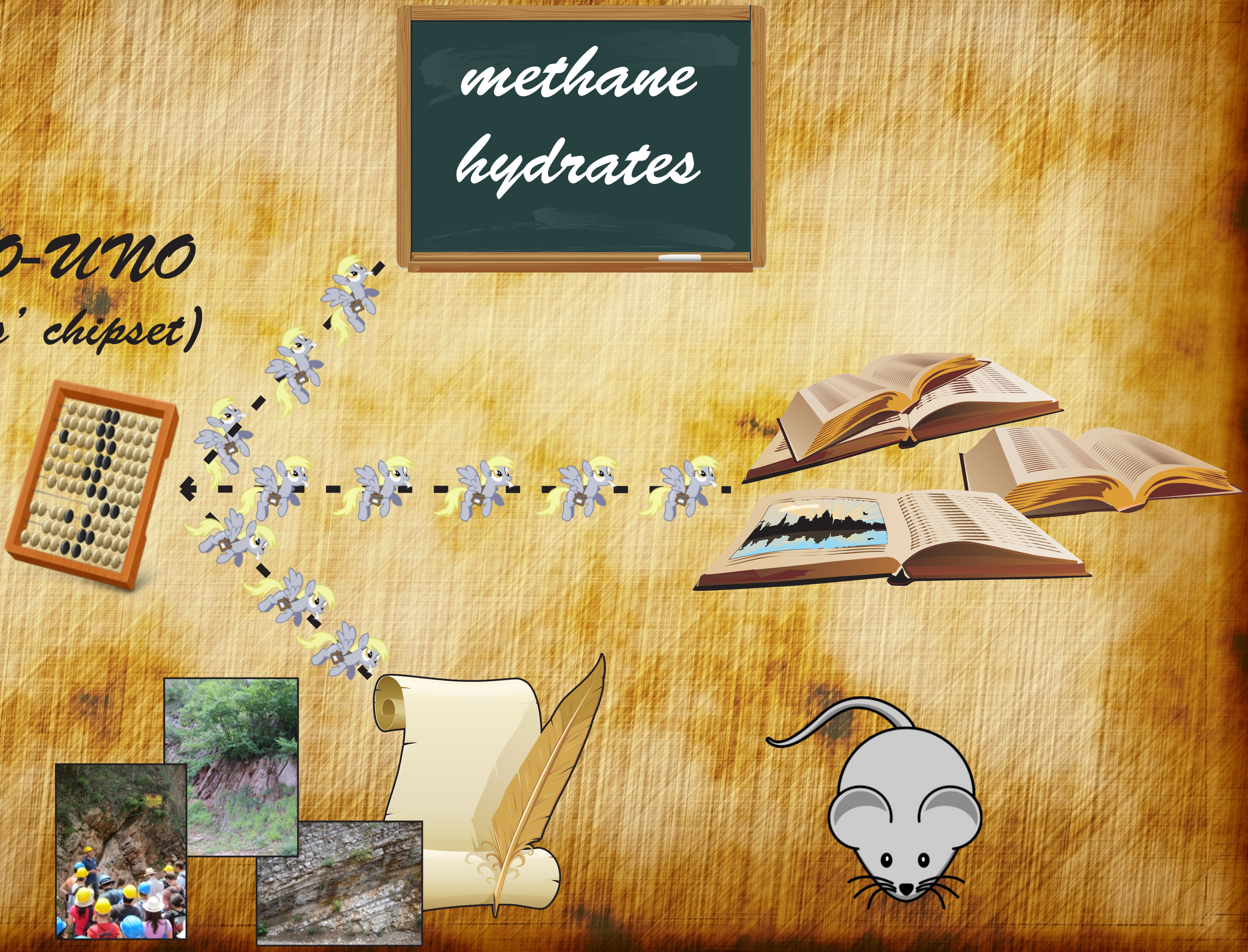
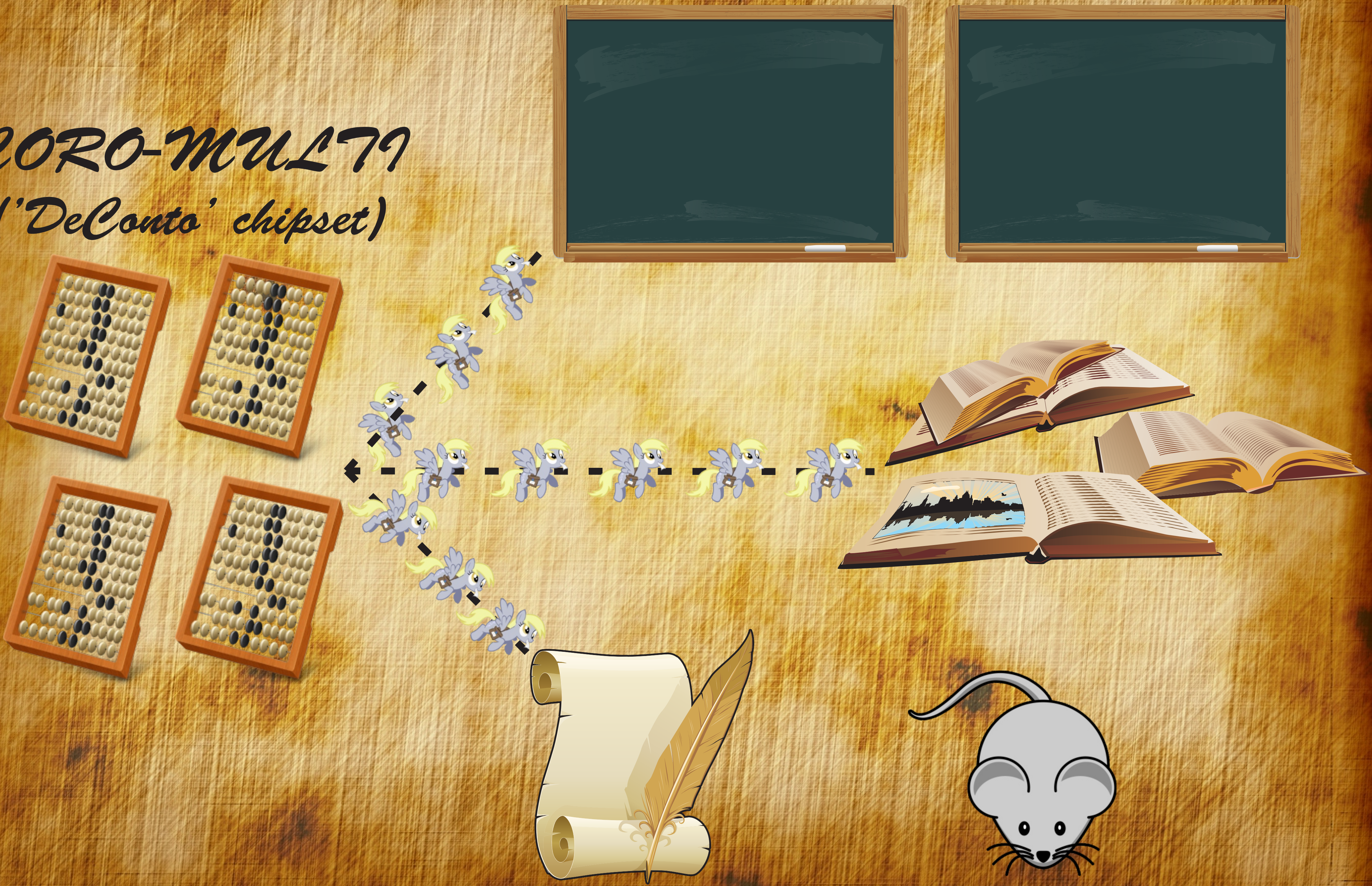


methane
hydrates

CORO-UNO
('Valdes' chipset)

A Device for Mechanical Algebra

CORO-MULTI
('DeConto' chipset)



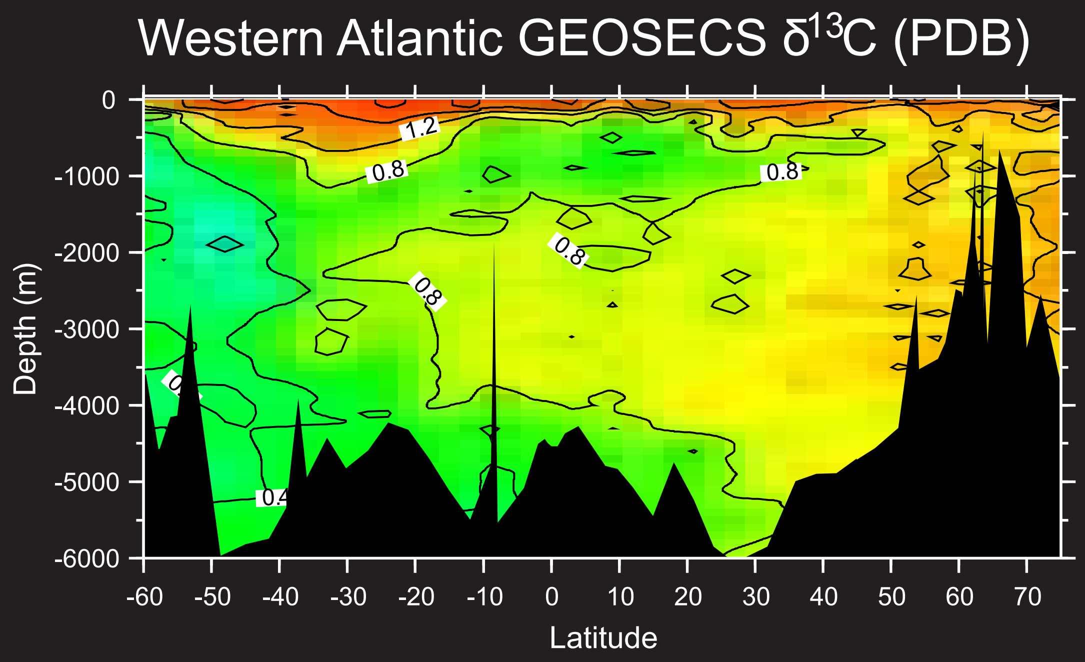
fun with models and data

(Biogeochemcial models & the art of (mis)using data)

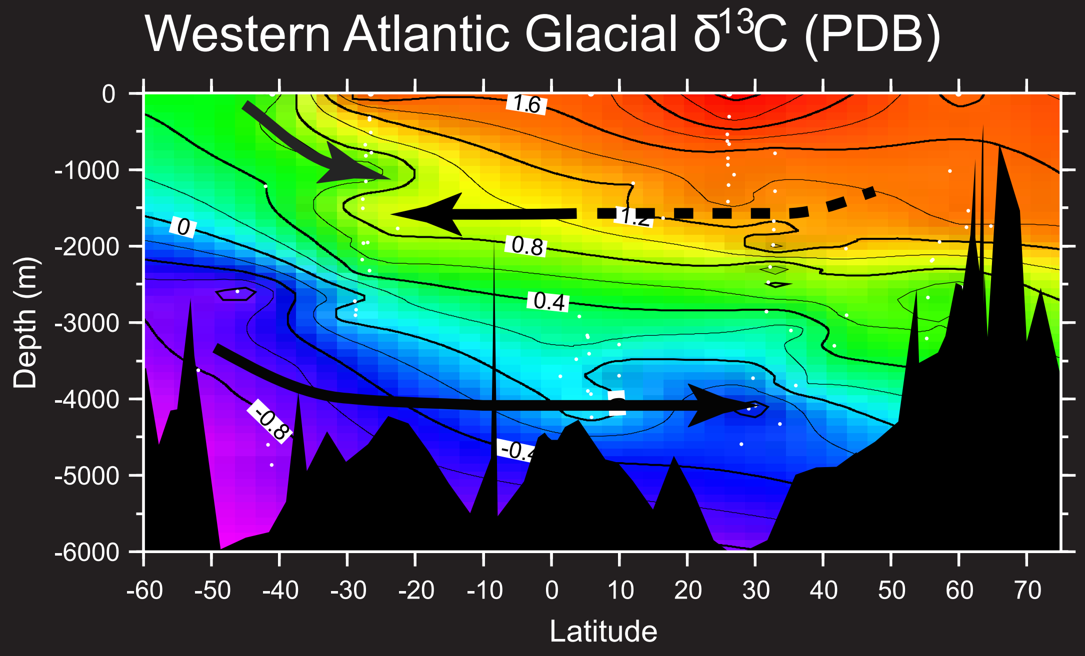
Andy Ridgwell

UCR
University of BRISTOL
THE ROYAL SOCIETY
CELEBRATING 350 YEARS
European Research Council
Established by the European Commission
erc

Meanwhile, in the ocean ...



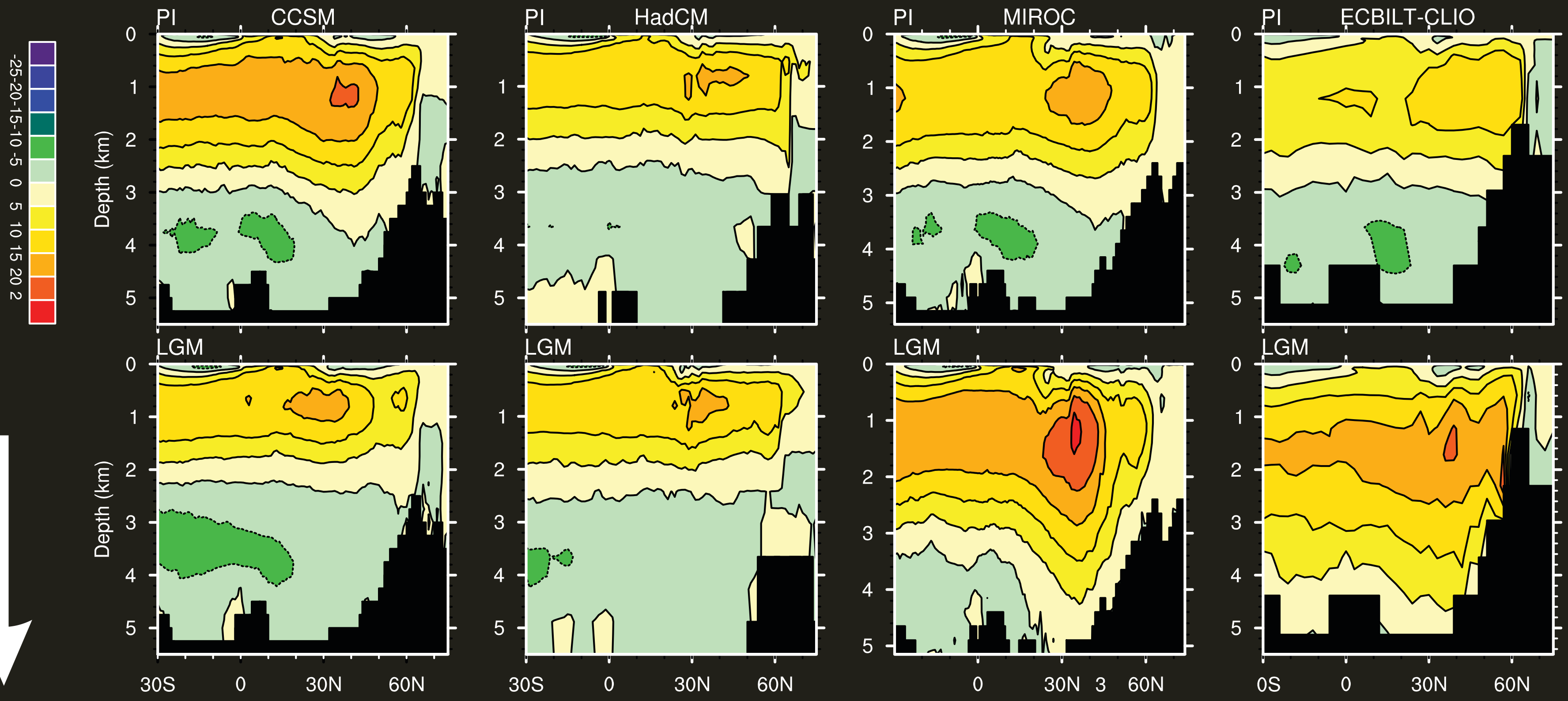
Western Atlantic GEOSECS $\delta^{13}C$ (PDB)



Western Atlantic Glacial $\delta^{13}C$ (PDB)

The distribution of $\delta^{13}C$ of ΣCO_2 in the modern western Atlantic [*Kroopnick*, 1985] vs. a recently updated glacial transect of $\delta^{13}C$ of ΣCO_2 for the western Atlantic Ocean basins [*Curry and Oppo*, 2005].

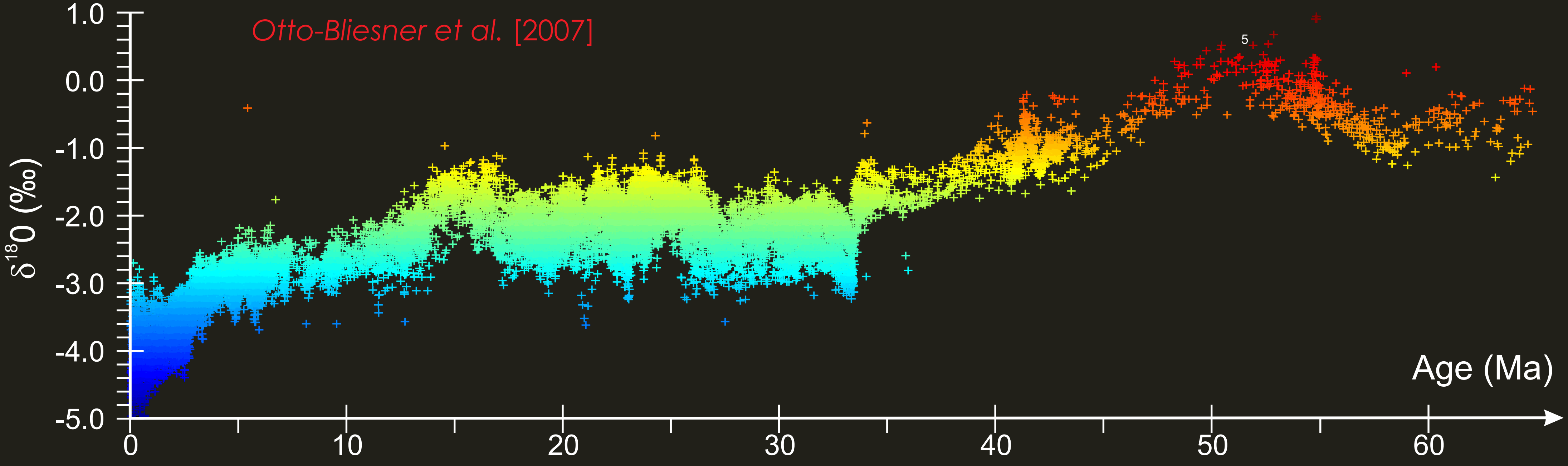
Spatial patterns, but still (n-1)-steps removed from the 'data'



modern

glacial

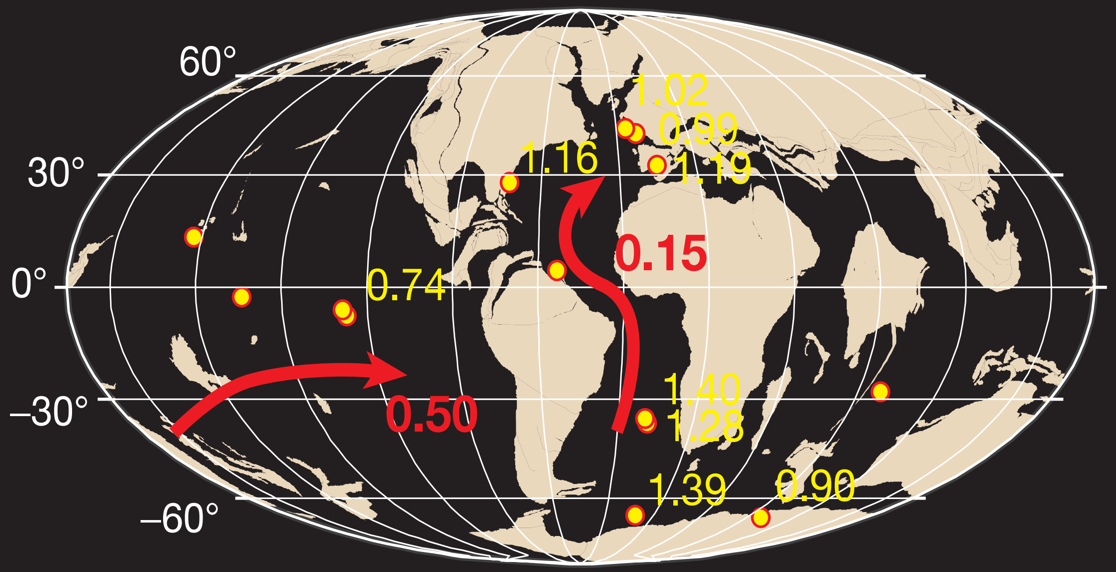
0.02 Ma



Otto-Bliesner et al. [2007]

Meanwhile, in the ocean ...

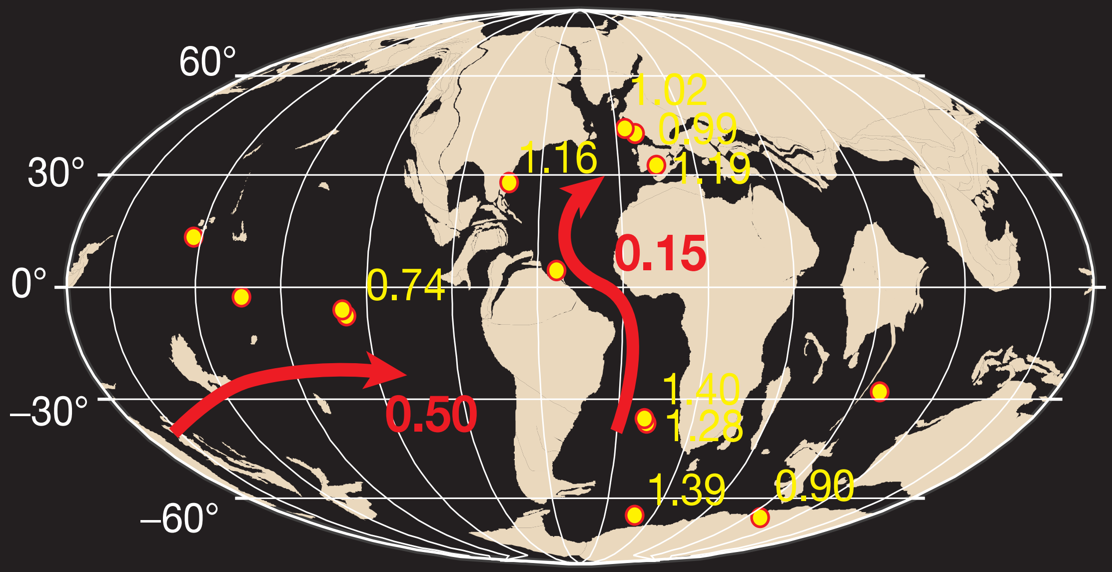
DATA:
Nunes and Norris [2006]



Late Paleocene
benthic δ^{13}C patterns

Down to 3-steps removed from the 'data'

DATA: *Nunes and Norris [2006]*

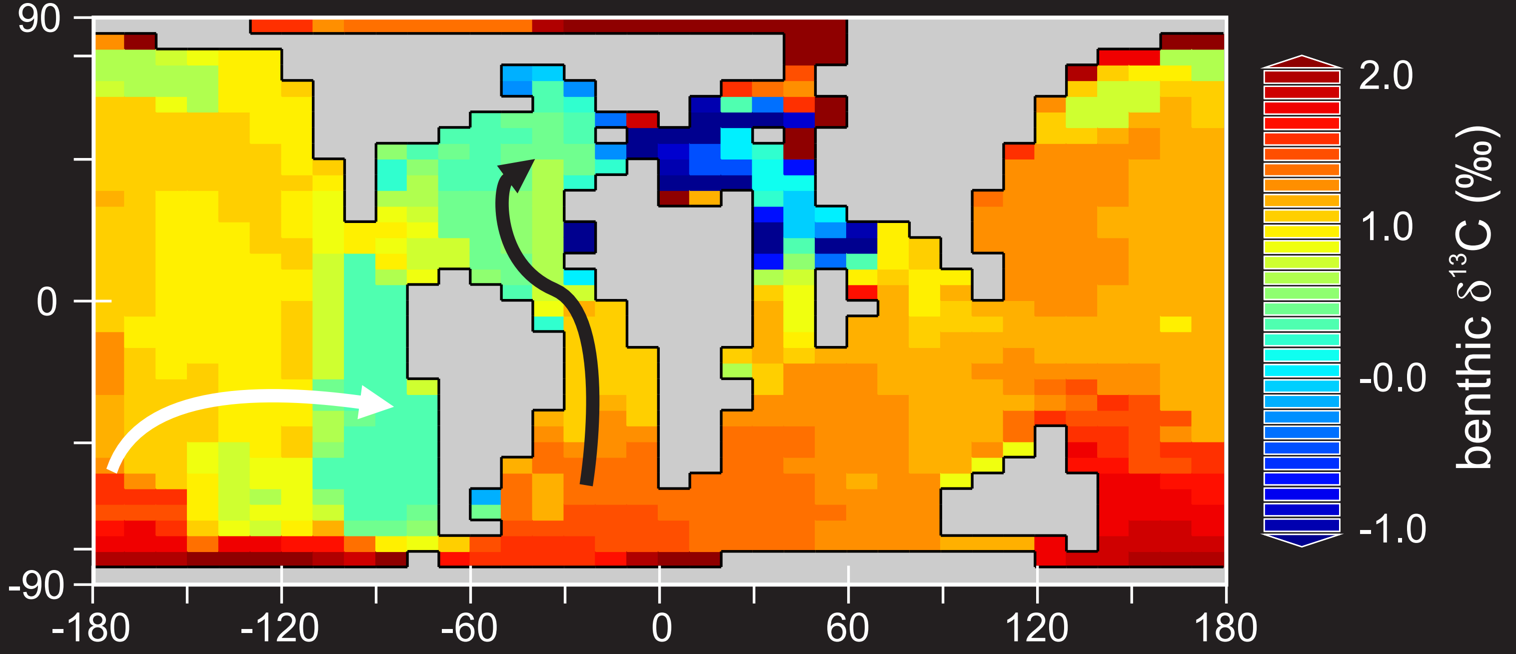


Late Paleocene benthic δ^{13}C patterns

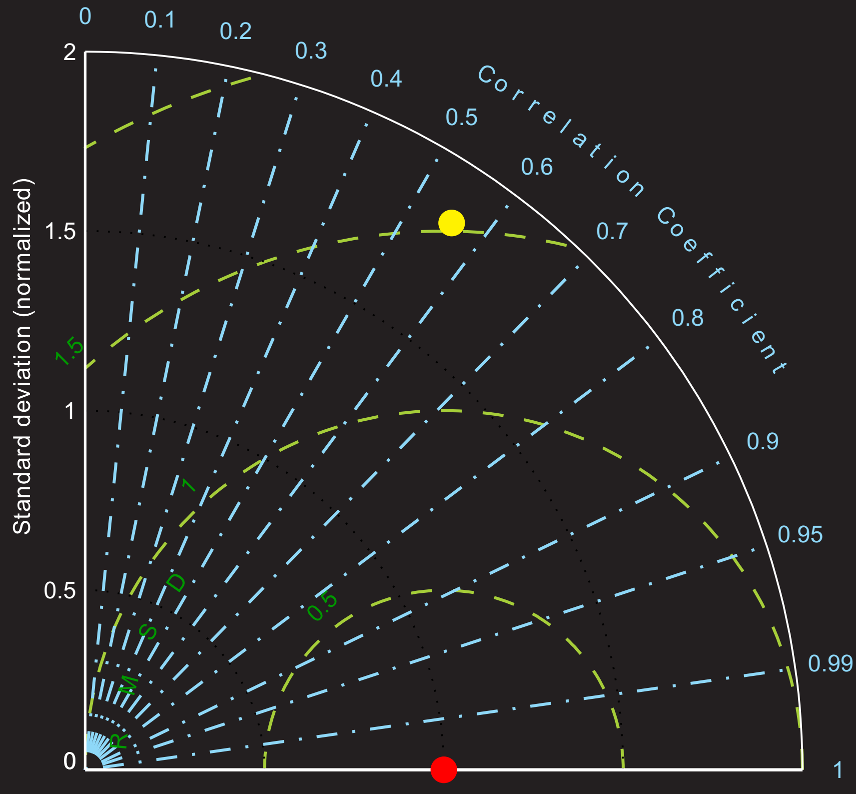
evaluation

Model-predicted gradients in benthic δ^{13}C (both direction and approximate magnitude) can be compared to available data-based reconstructions.

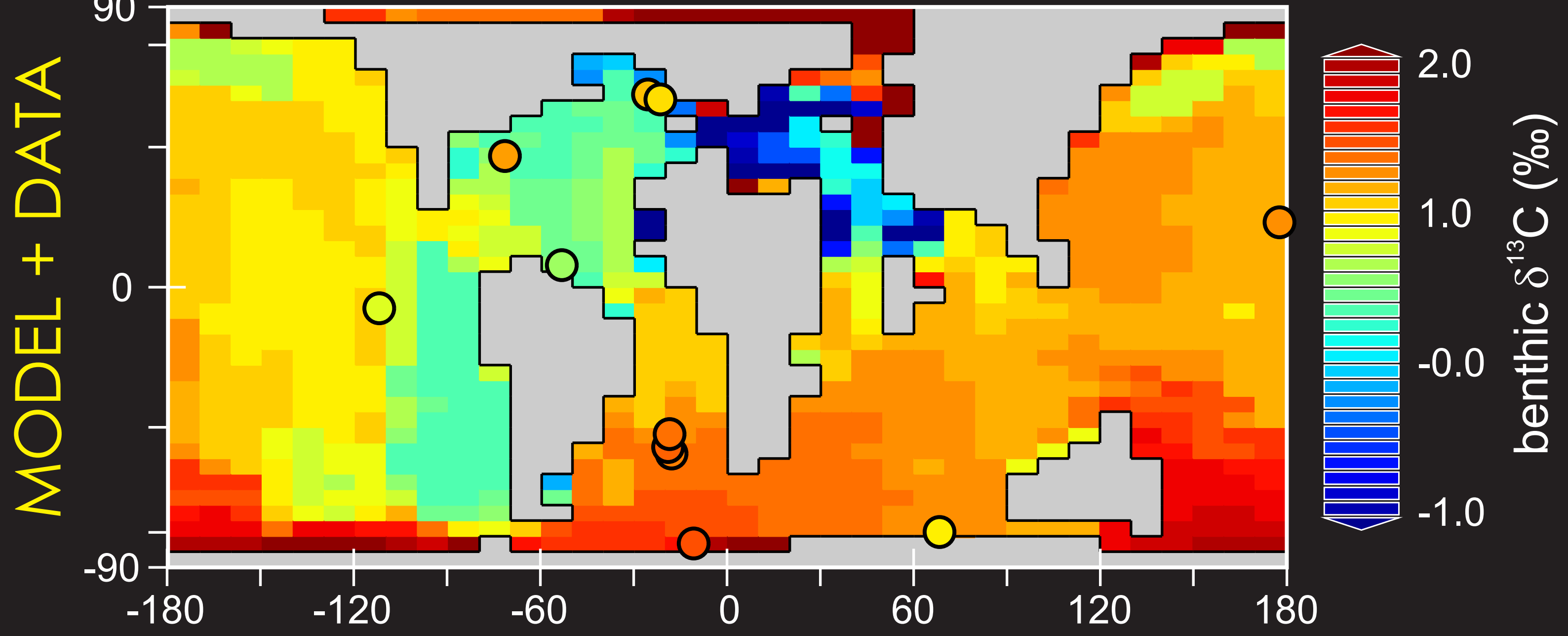
MODEL



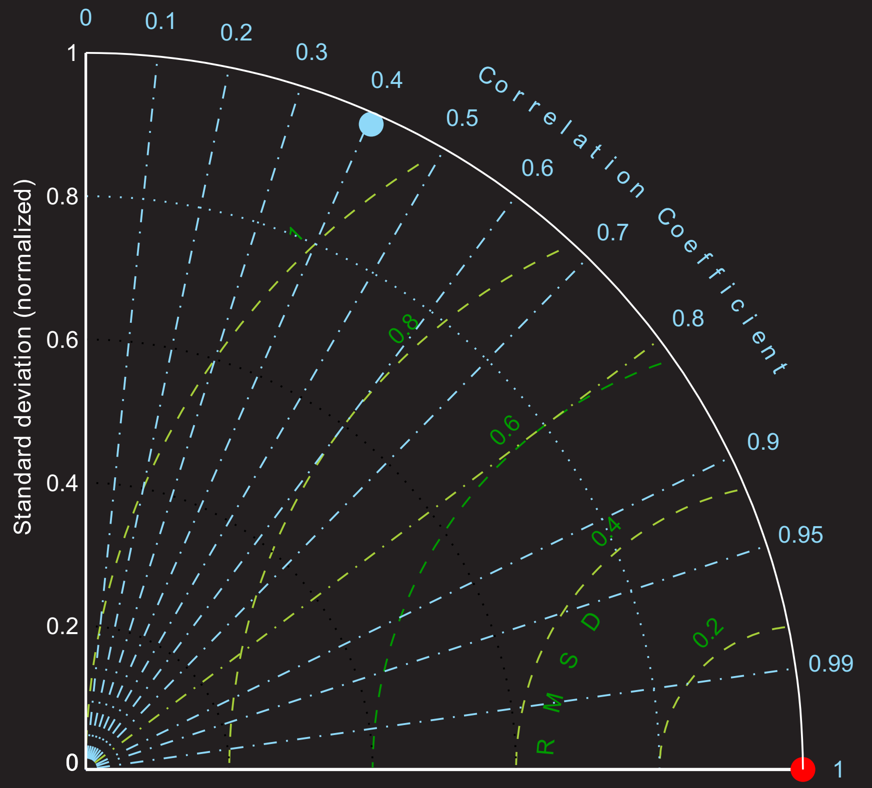
3-steps removed from the 'data'

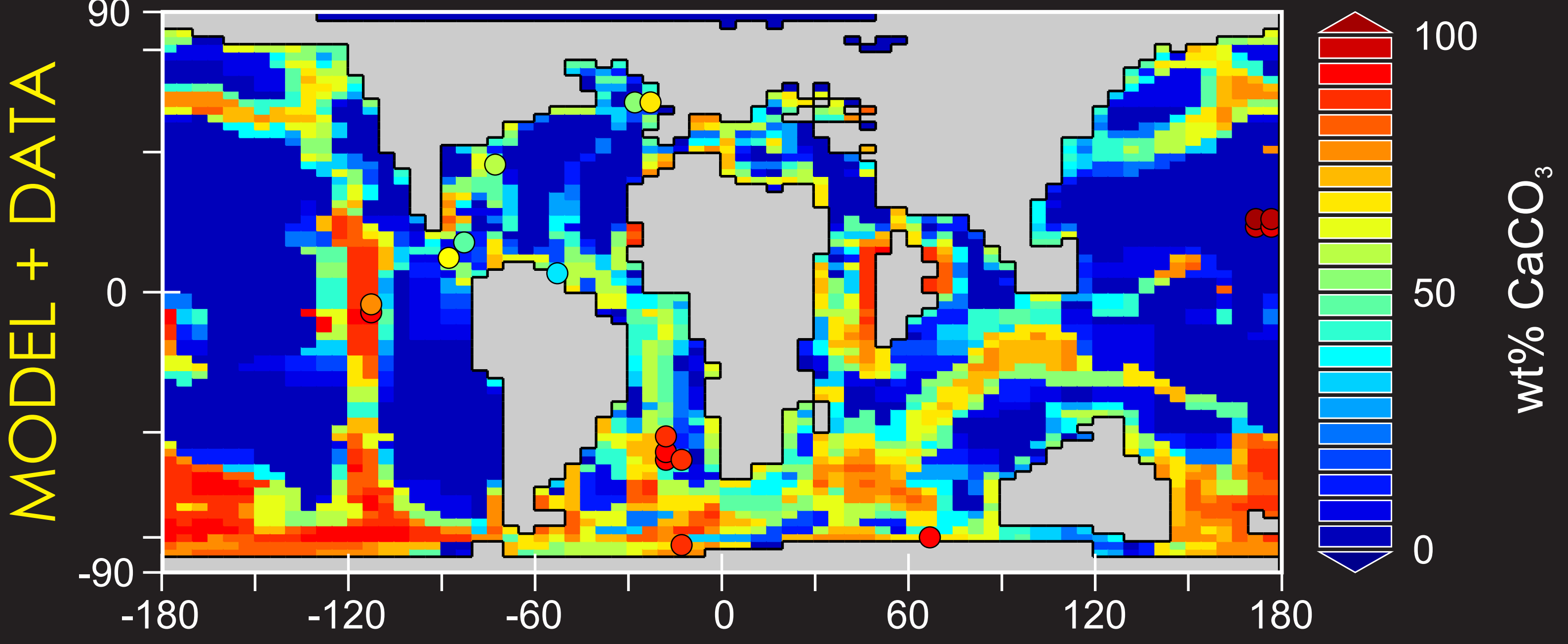


Model-predicted benthic $\delta^{13}C$ can be assessed statistically vs. observations by e.g., 'Taylor diagrams'

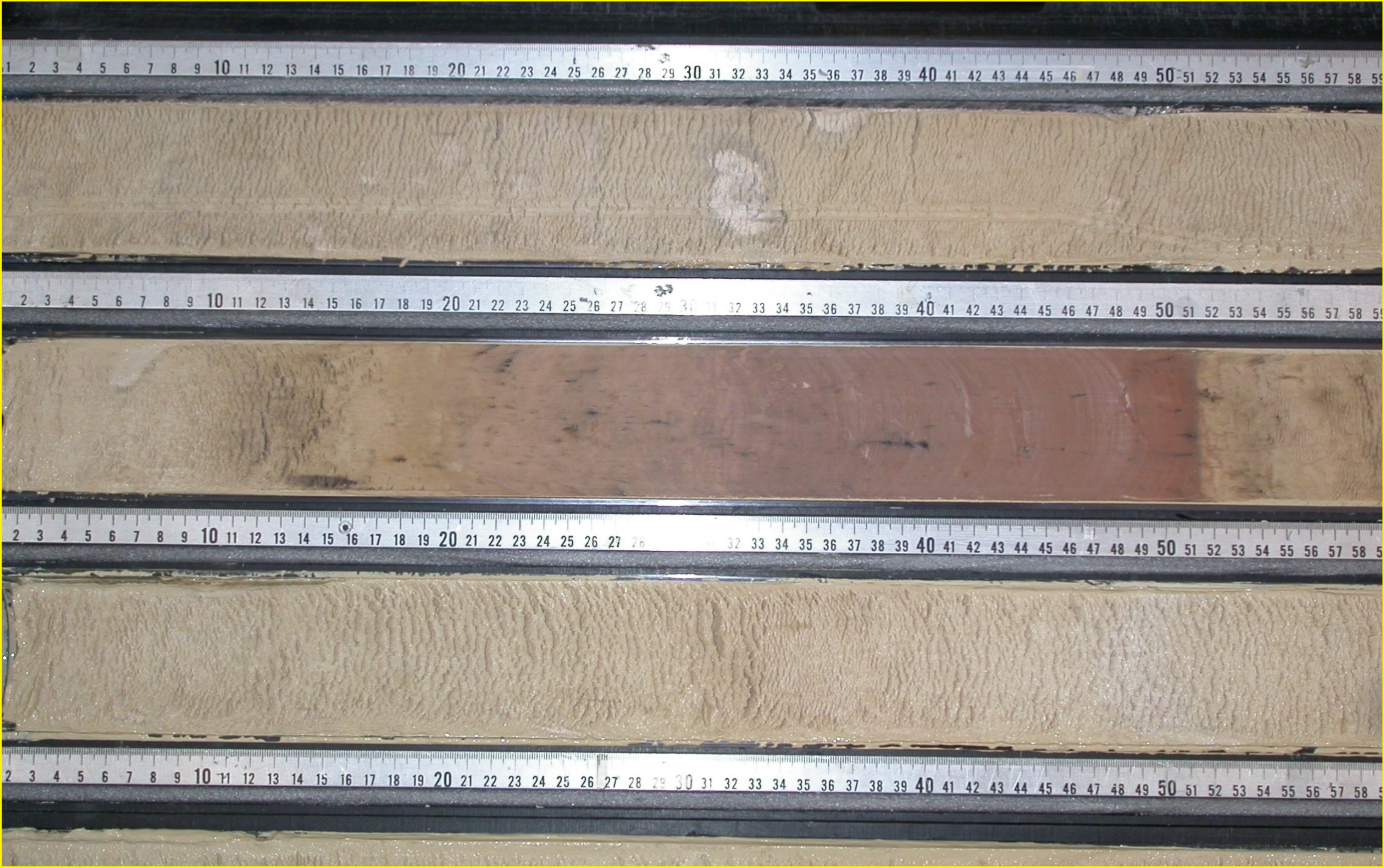


2-steps removed from the 'data'



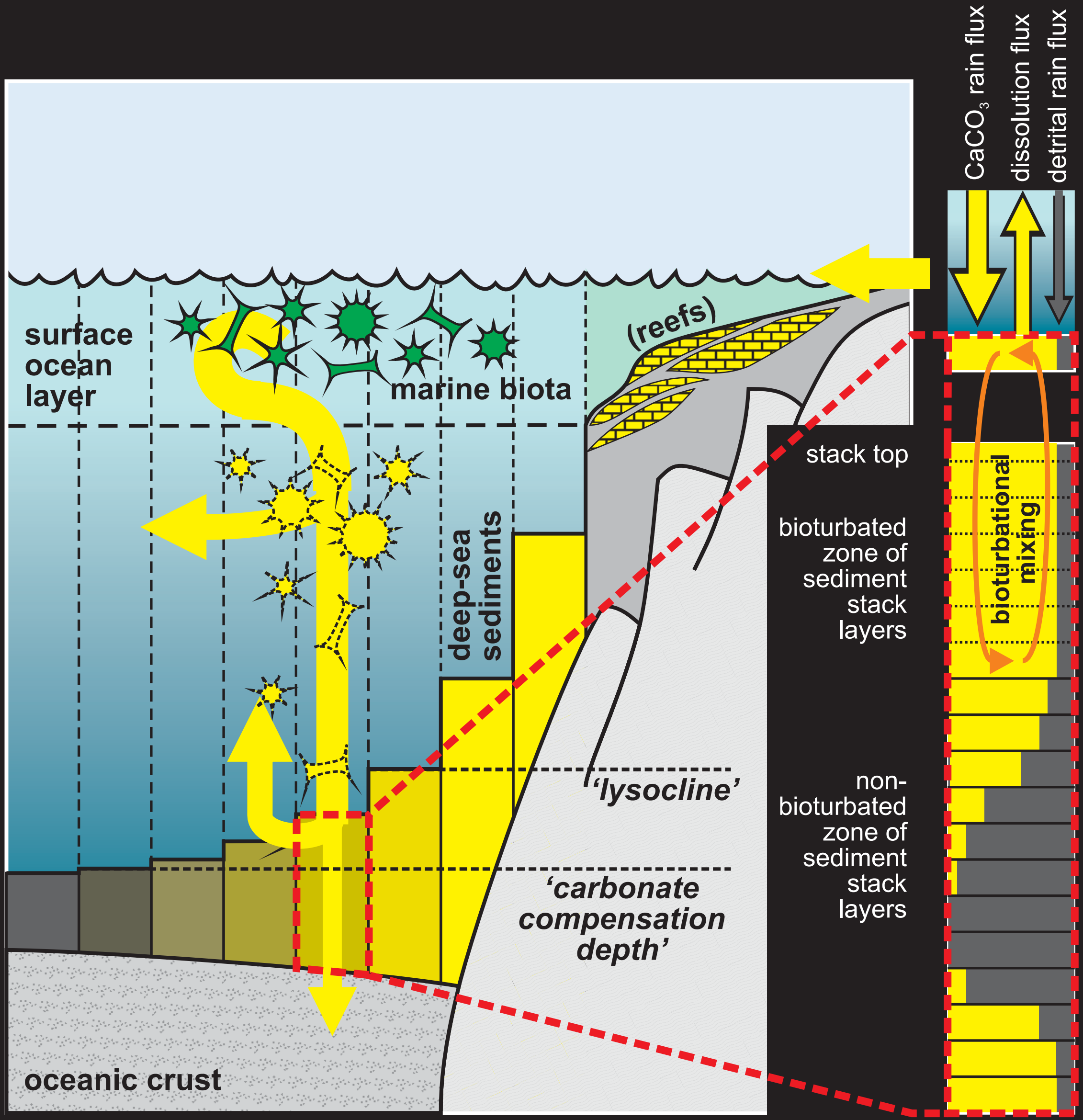


The data ...

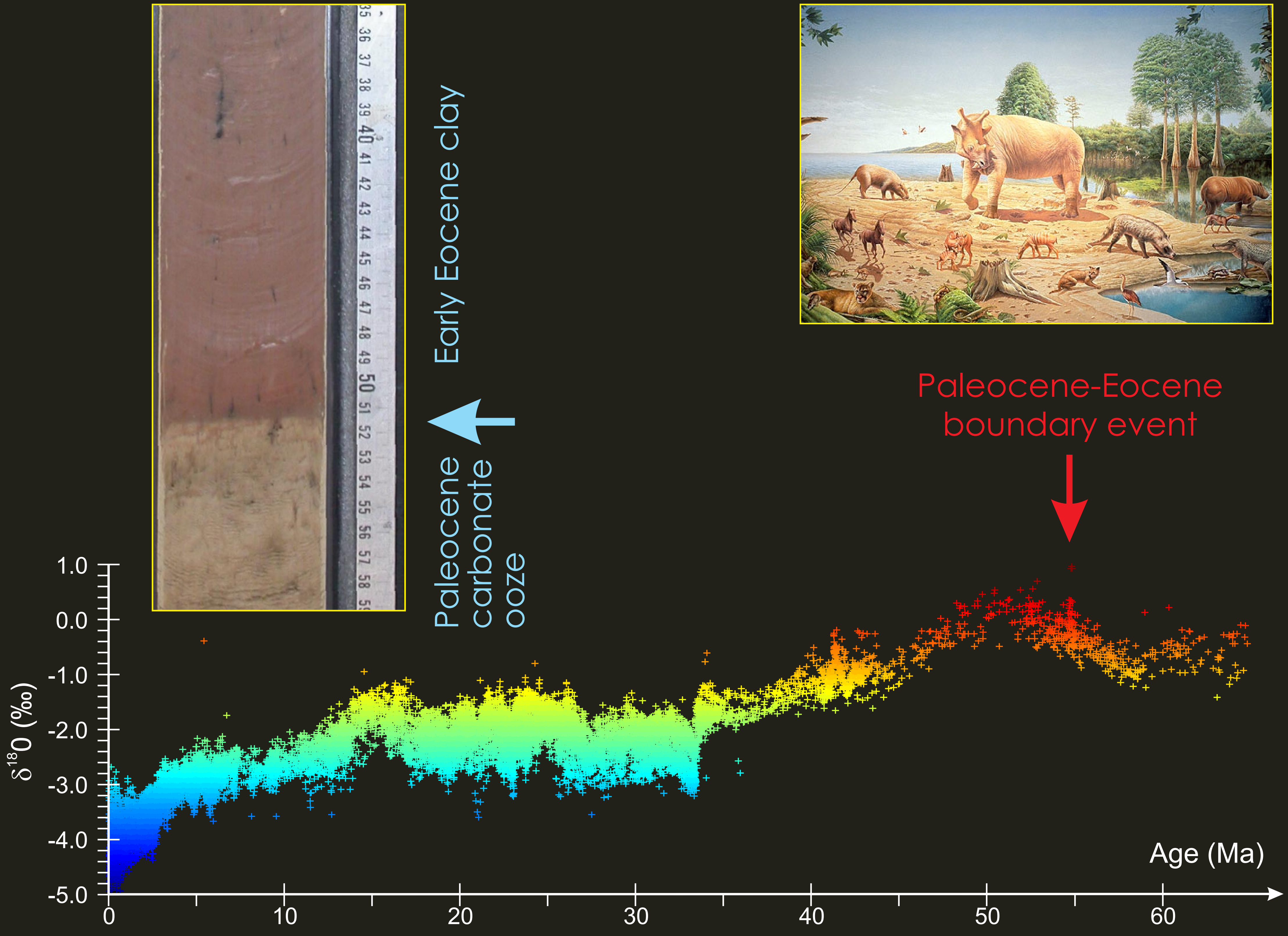


Sediments spanning the Palaeocene-Eocene boundary recovered from ODP Leg 208 (Walvis Ridge)
Picture courtesy of Daniela Schmidt (University of Bristol)

How to get to 1-step removed from the 'data'



(Vital effects can be parameterized as e.g. planktic (or benthic) $CaCO_3$ is formed. However, still no consideration of multiple fractions of $CaCO_3$ with different dissolution susceptibility or diagenesis in general.)

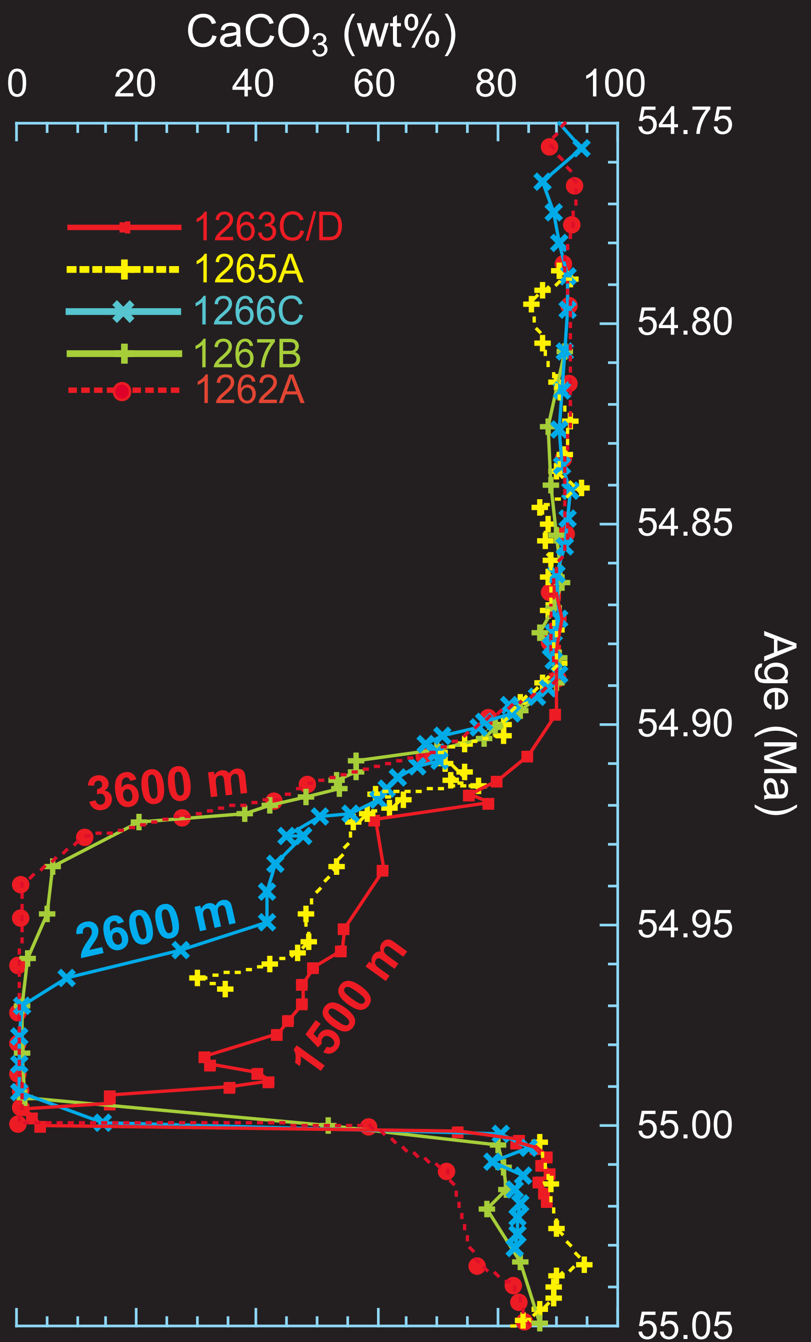


Early Eocene clay

Paleocene carbonate ooze

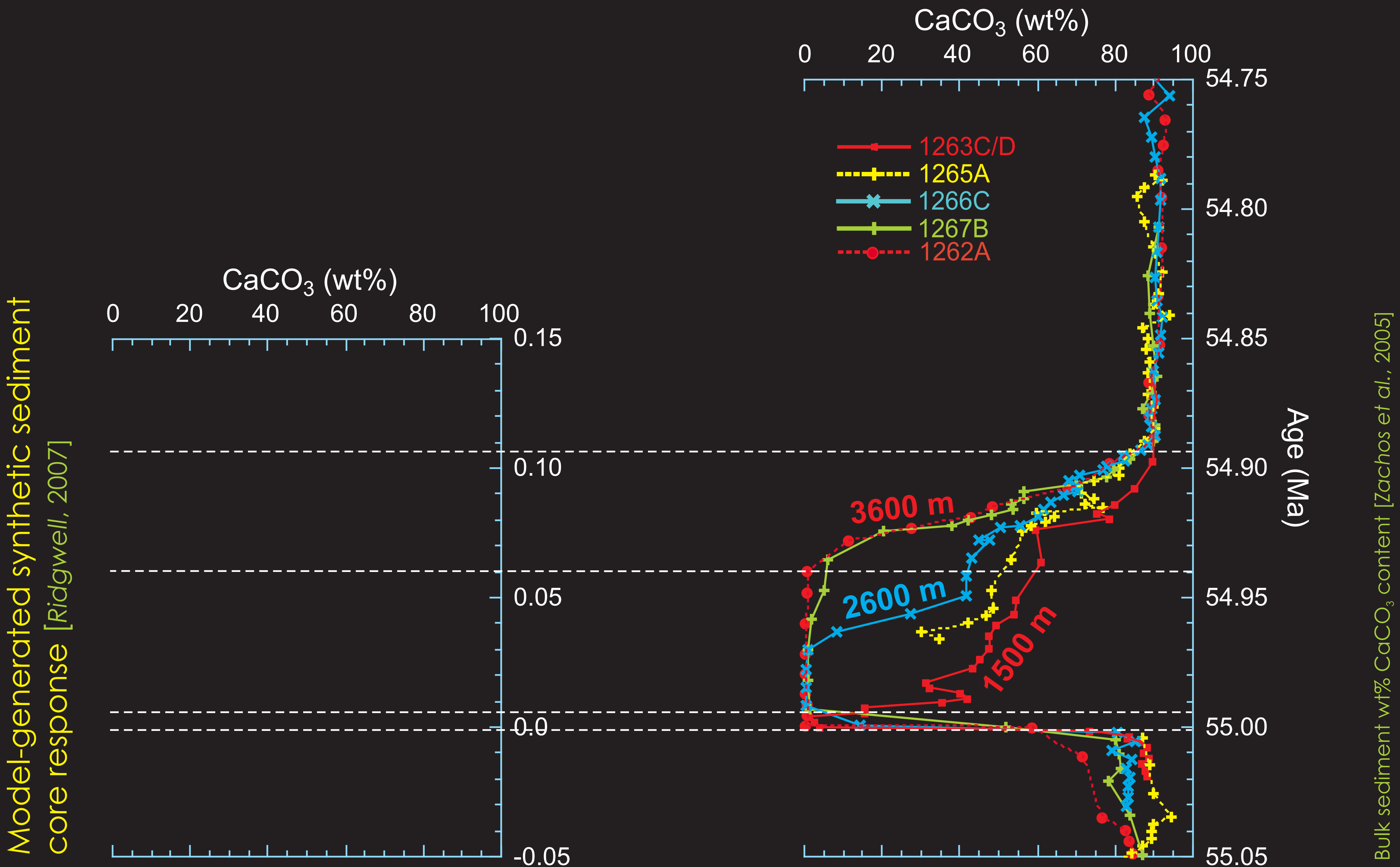
Paleocene-Eocene boundary event

Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'



Bulk sediment wt% $CaCO_3$ content [*Zachos et al.*, 2005]

Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'



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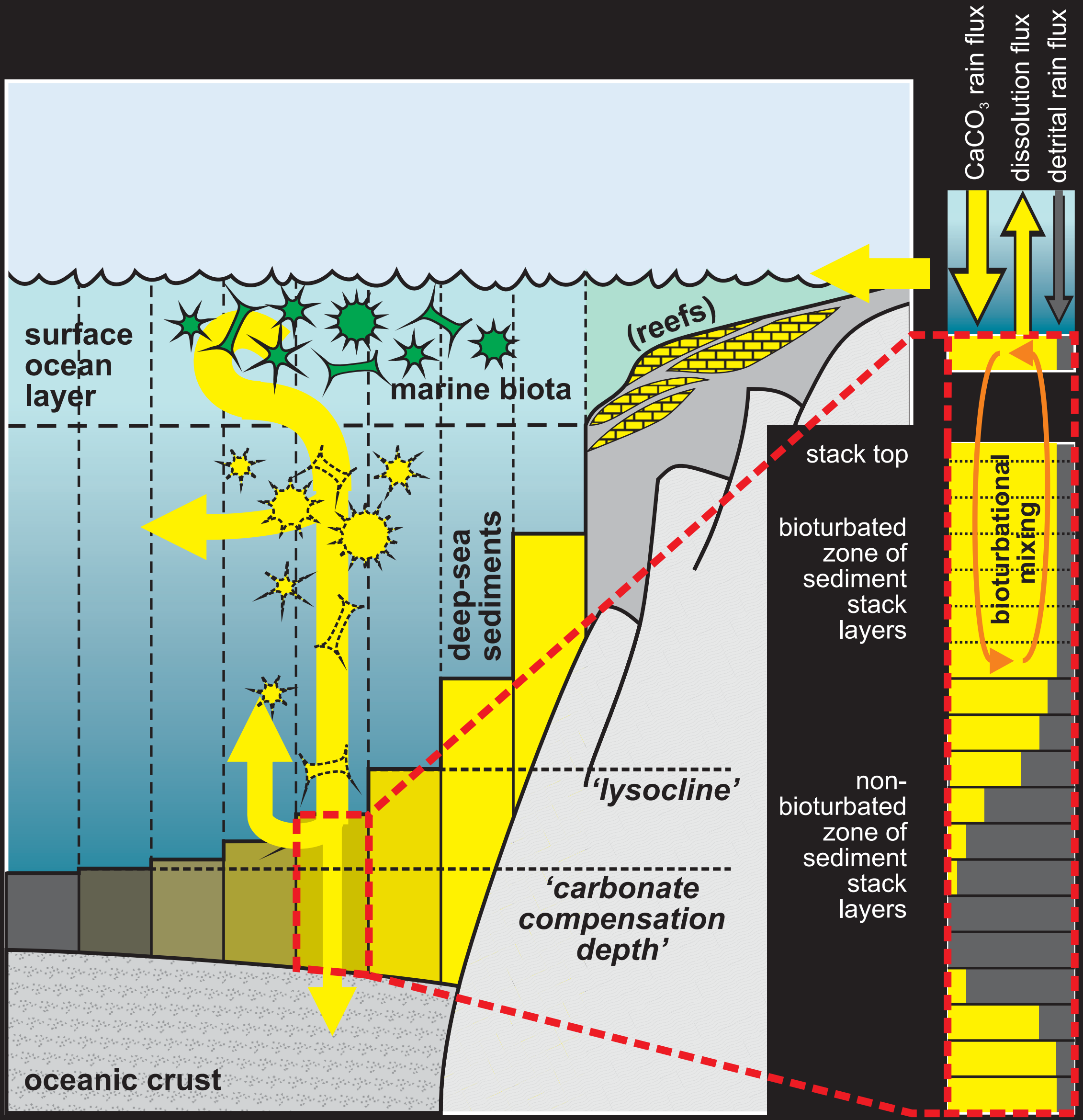

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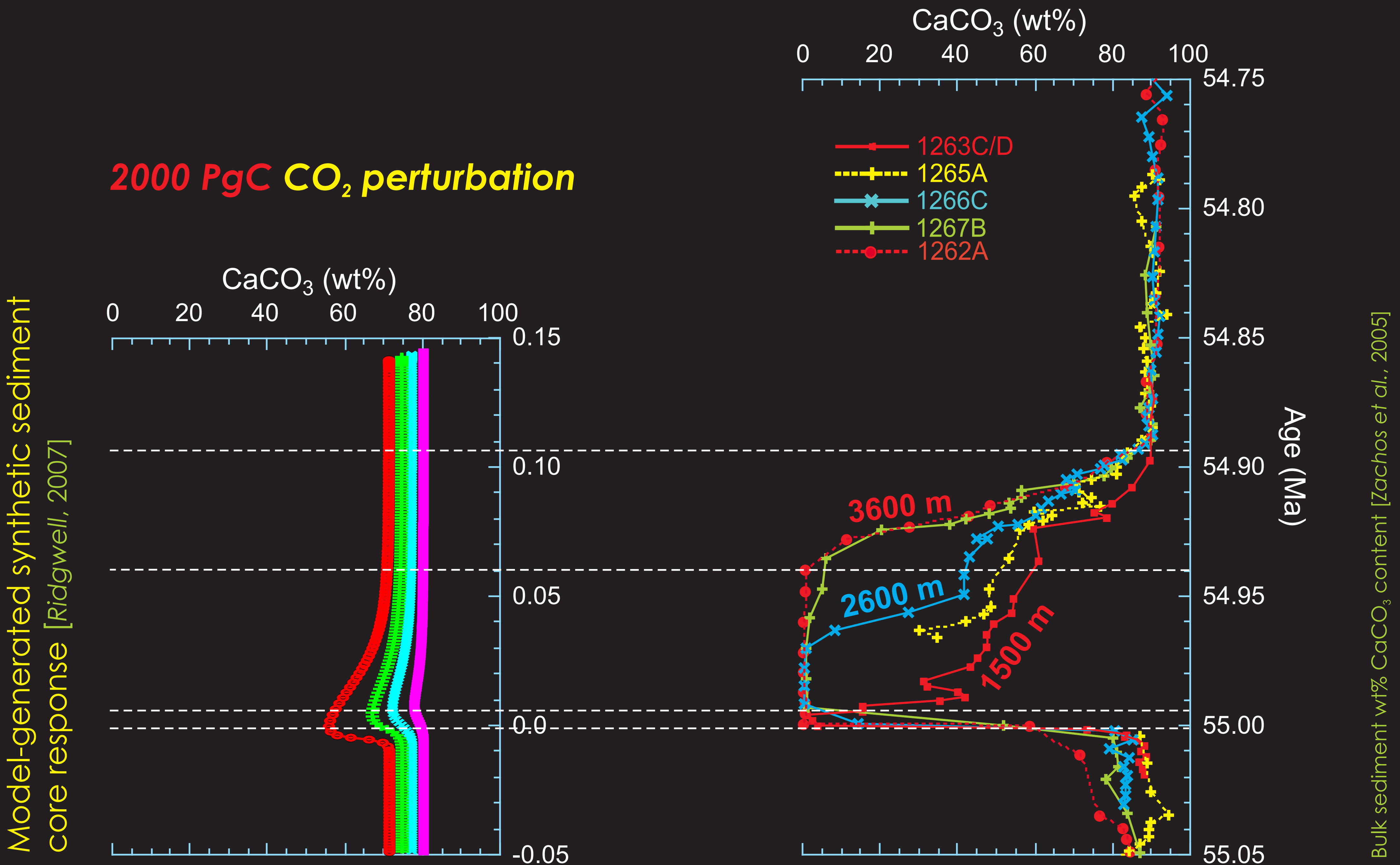
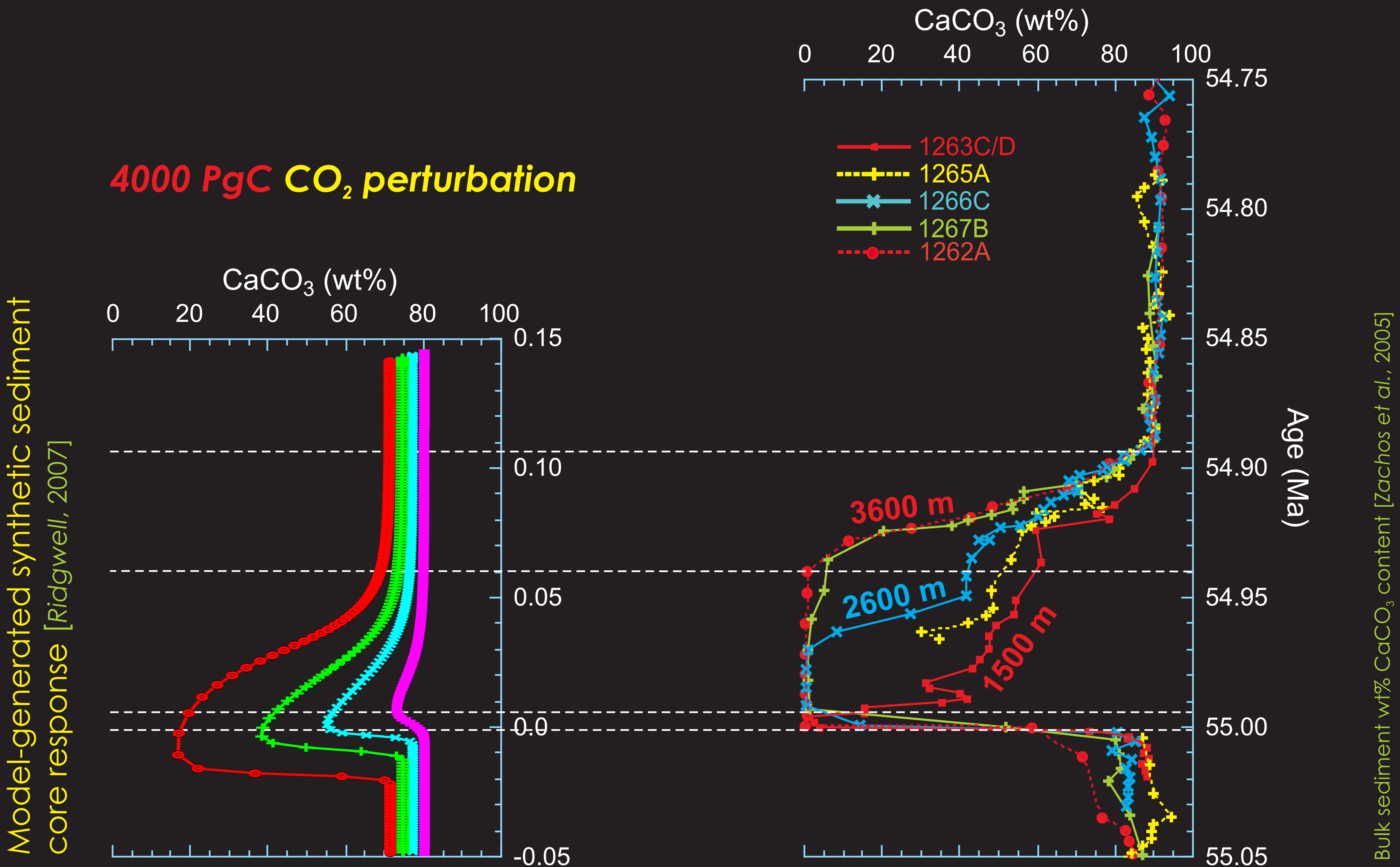


2000 PgC CO_2 perturbation

Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'

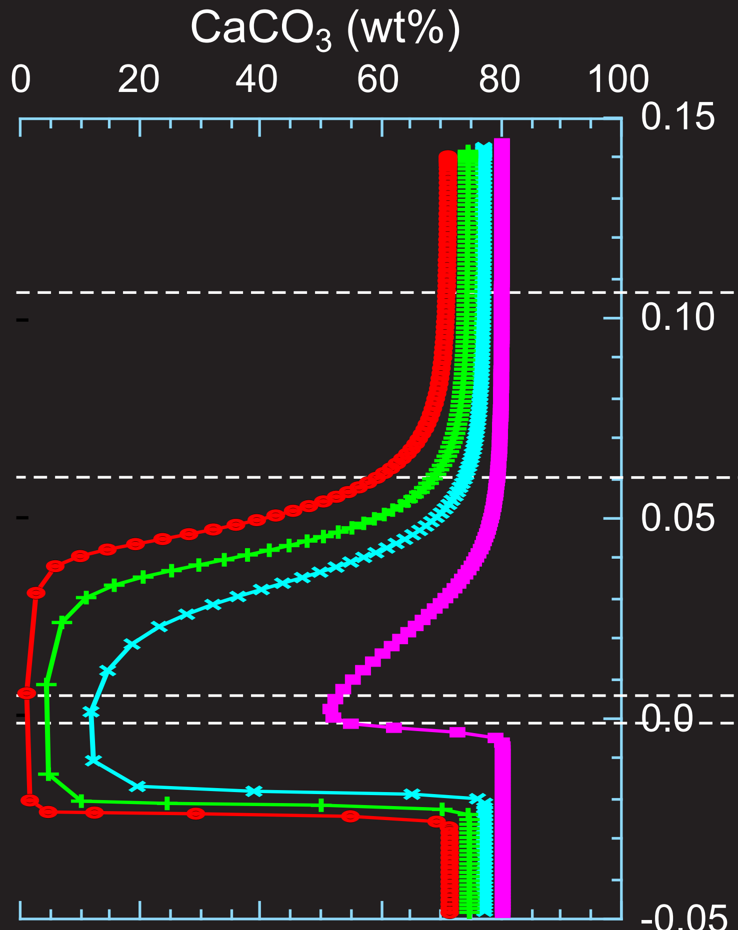


4000 PgC CO_2 perturbation

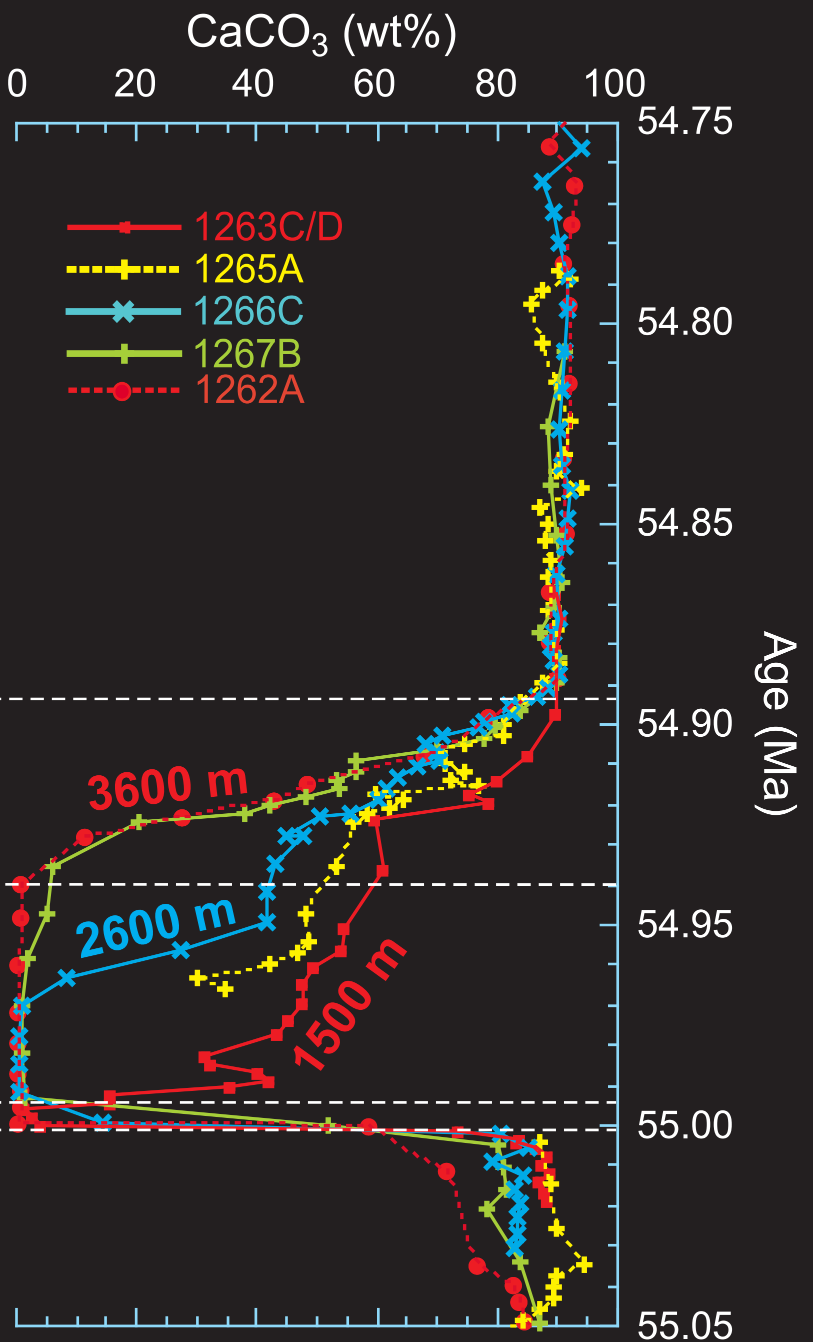
Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'

6000 PgC CO_2 perturbation

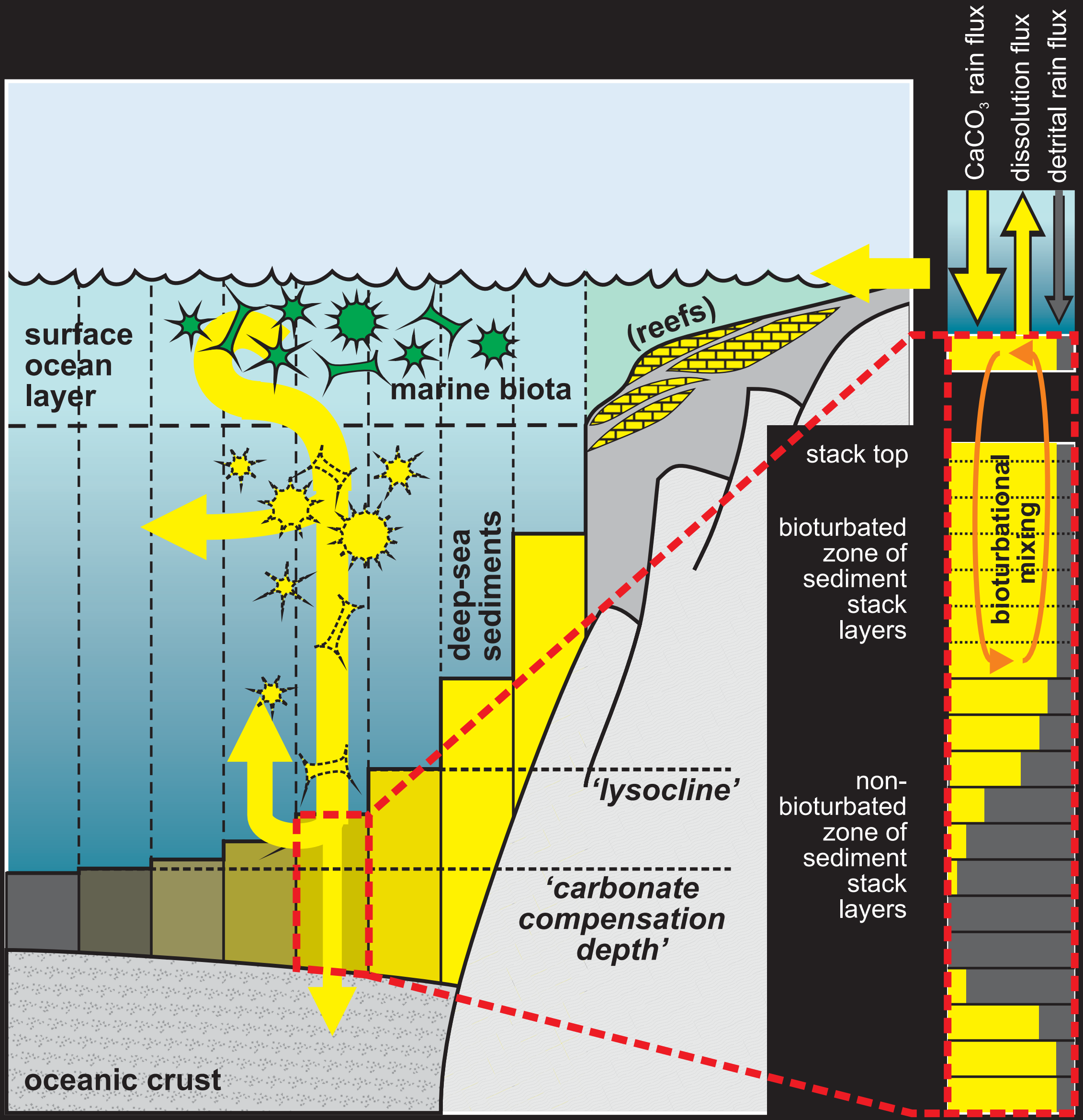
Model-generated synthetic sediment core response [*Ridgwell*, 2007]

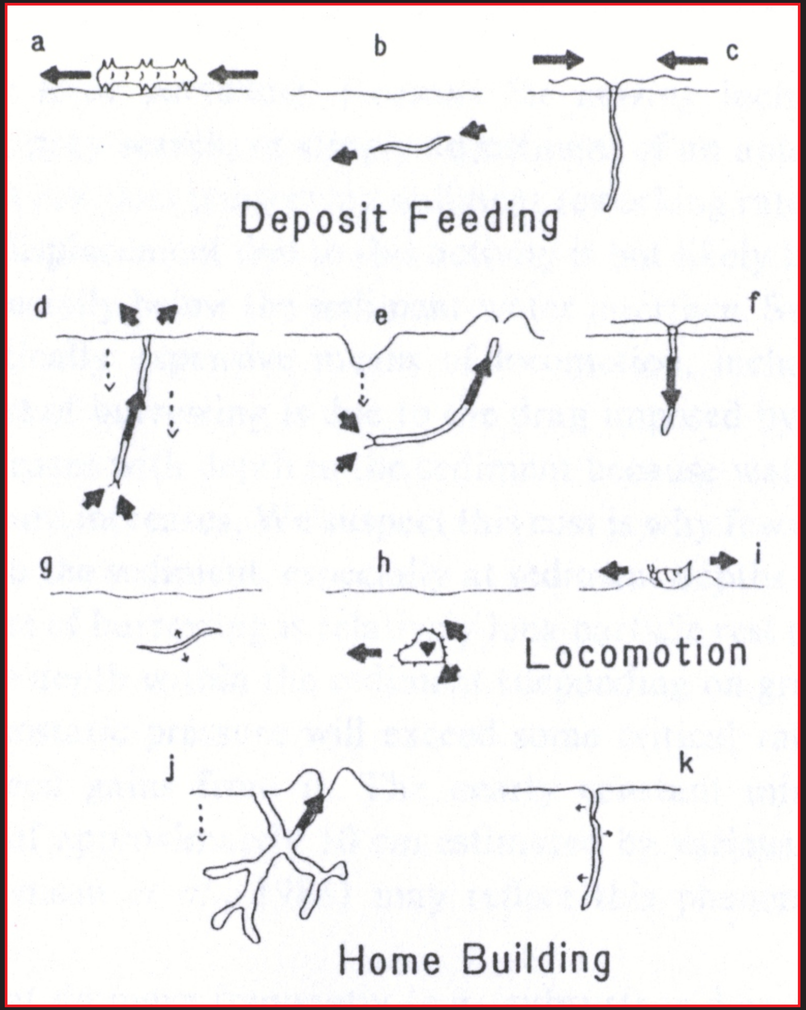


Bulk sediment wt% $CaCO_3$ content [*Zachos et al.*, 2005]



Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'

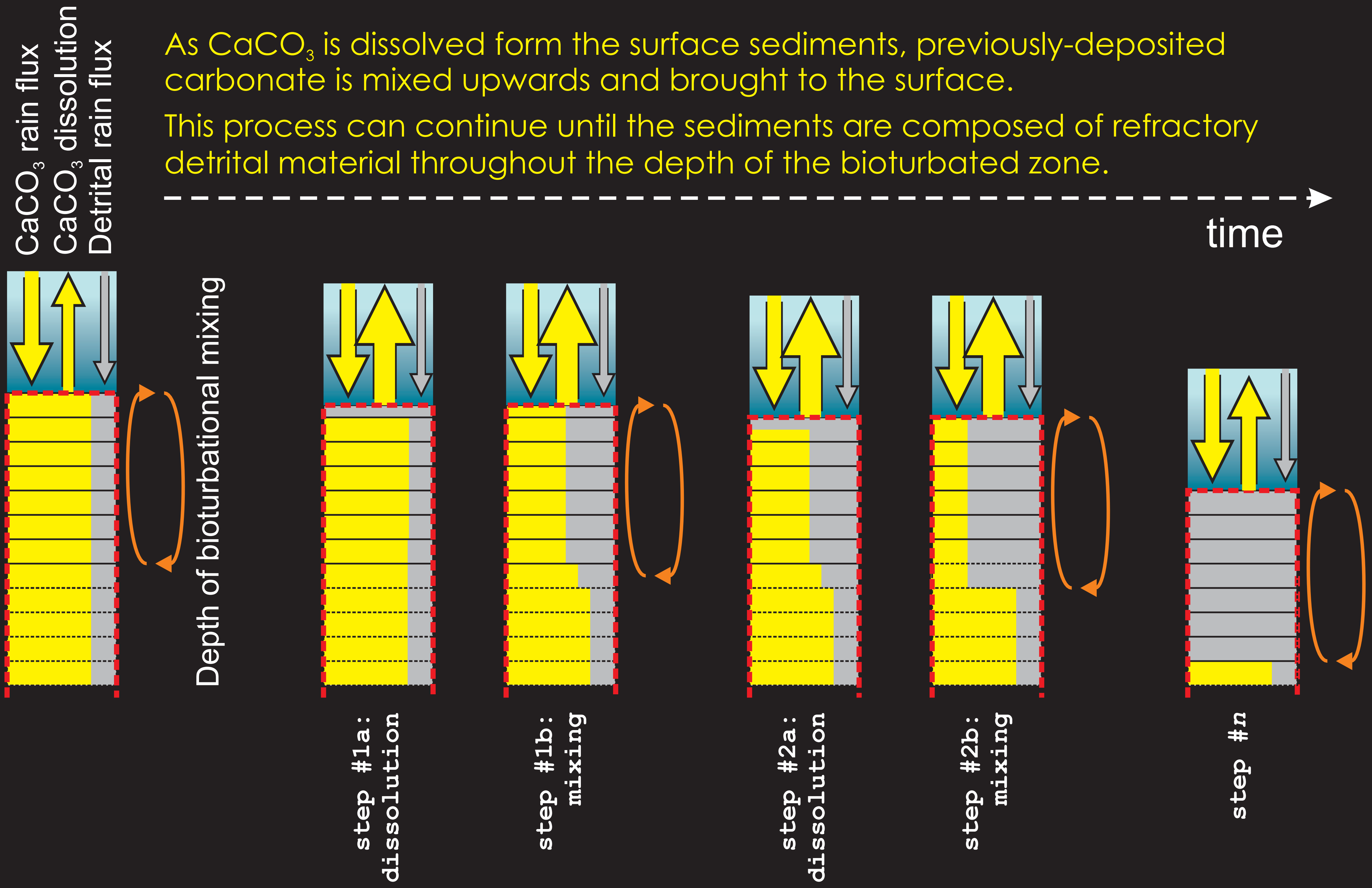




The role of 'bioturbation' (burrowing/injesting/filtering bugs) in sedimentary $CaCO_3$ dissolution

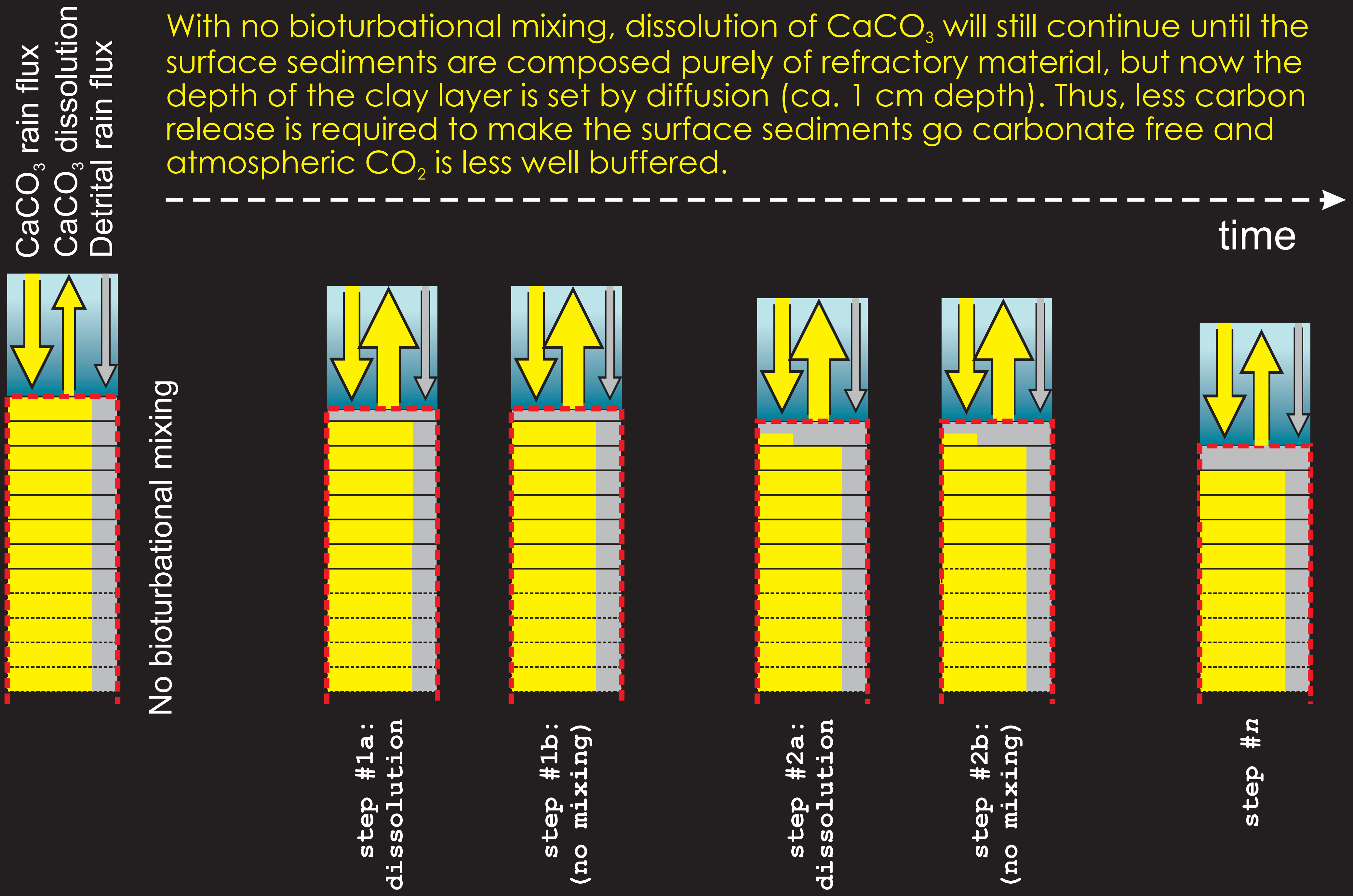
As $CaCO_3$ is dissolved form the surface sediments, previously-deposited carbonate is mixed upwards and brought to the surface.

This process can continue until the sediments are composed of refractory detrital material throughout the depth of the bioturbated zone.



The role of 'bioturbation' (burrowing/injesting/filtering bugs) in sedimentary $CaCO_3$ dissolution

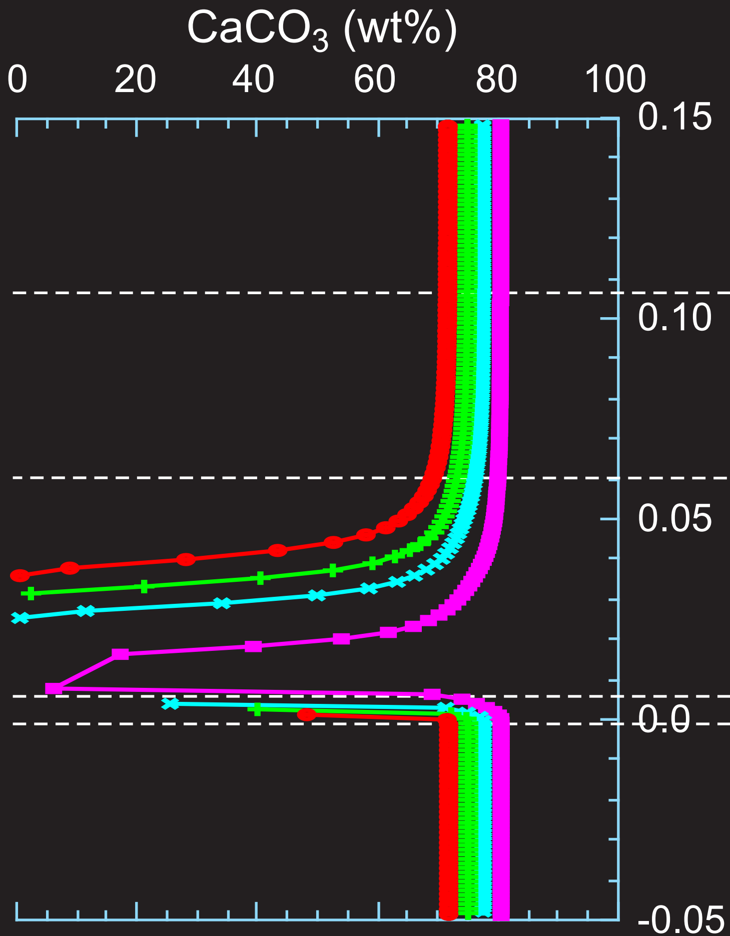
With no bioturbational mixing, dissolution of $CaCO_3$ will still continue until the surface sediments are composed purely of refractory material, but now the depth of the clay layer is set by diffusion (ca. 1 cm depth). Thus, less carbon release is required to make the surface sediments go carbonate free and atmospheric CO_2 is less well buffered.

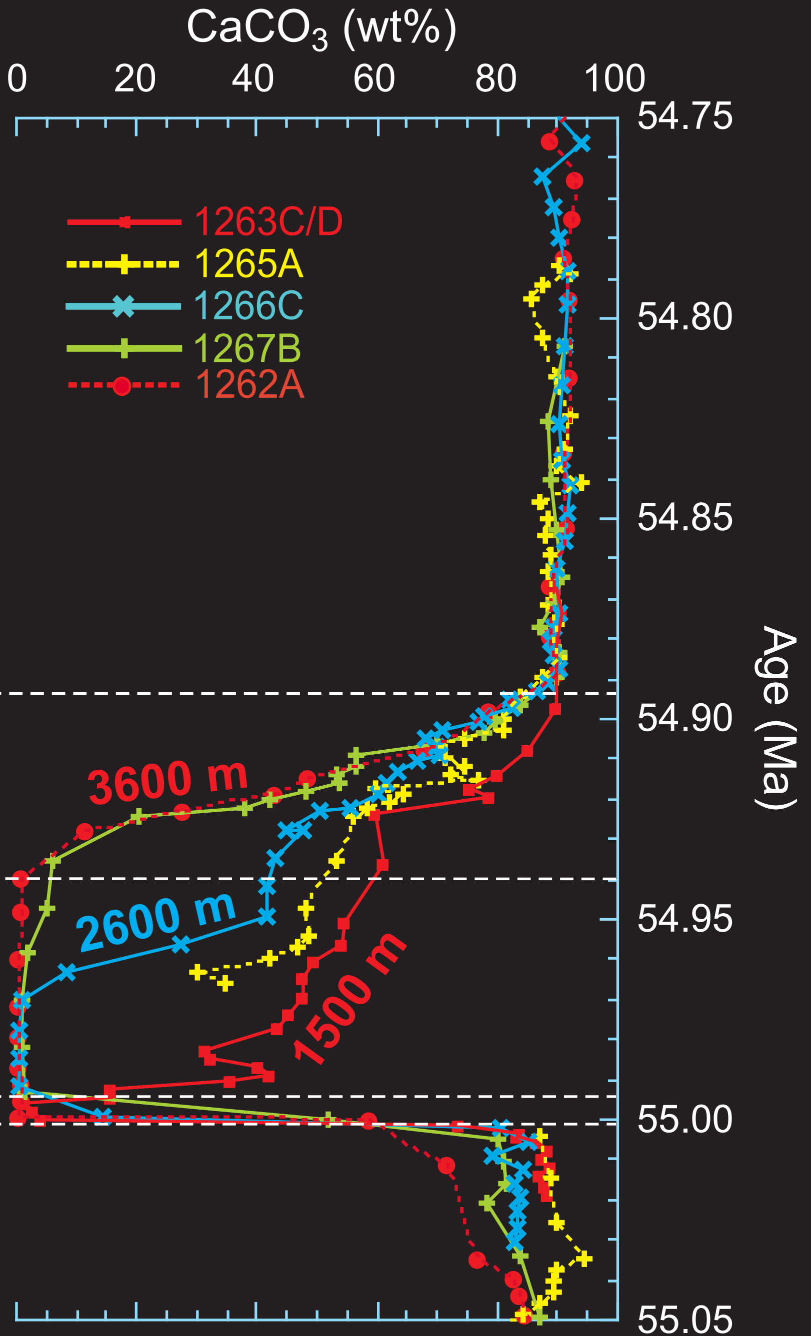


Exploring the consequences of CO_2 release through simulating the marine geological record: 'trial and error'

4000 PgC CO_2 perturbation
no bioturbation

Model-generated synthetic sediment core response [*Ridgwell*, 2007]

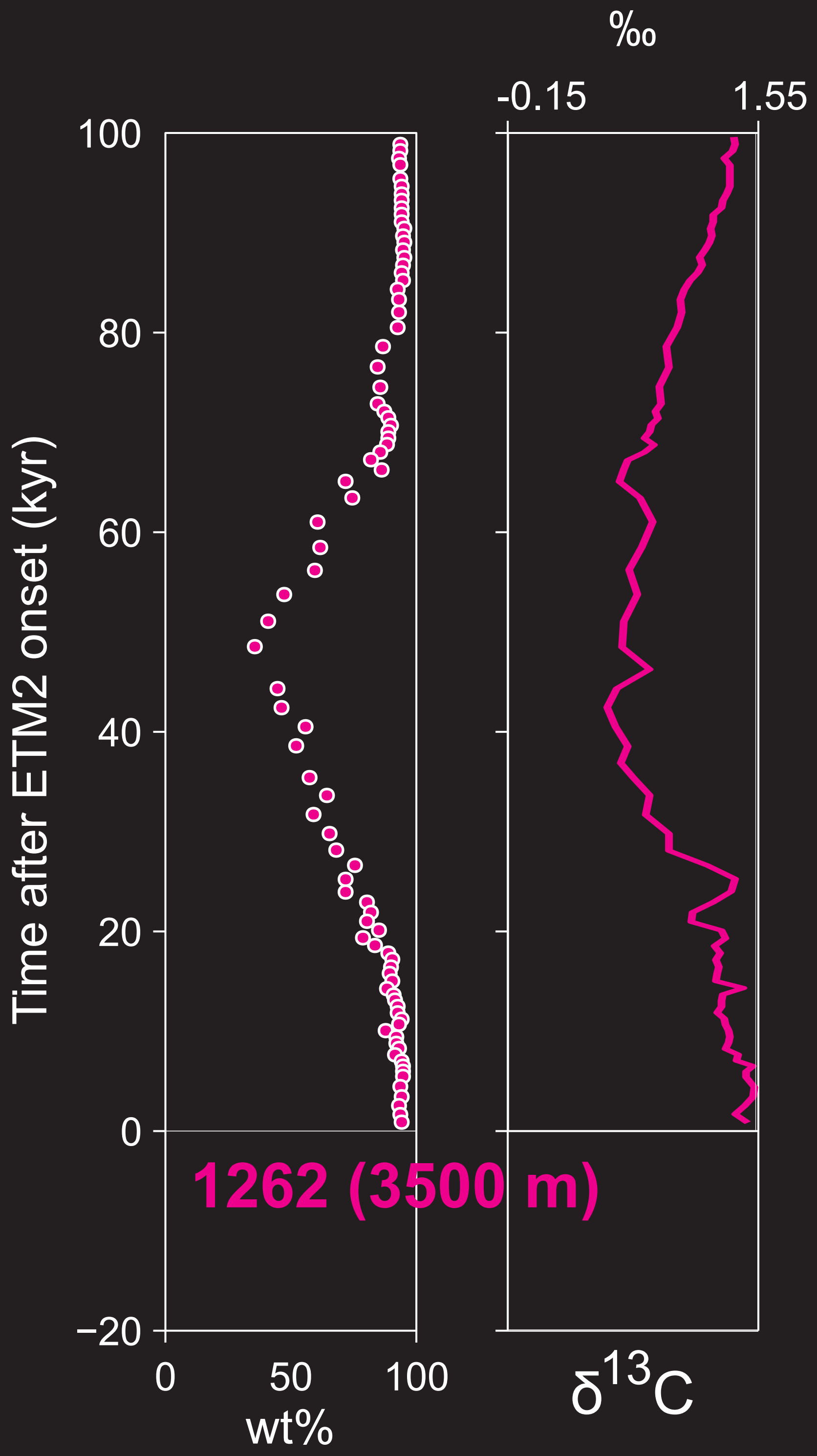




Bulk sediment wt% $CaCO_3$ content [*Zachos et al.*, 2005]

Fun with models and data: ETM2

Stap et al. [2010]; *Jennions et al.* [submitted]



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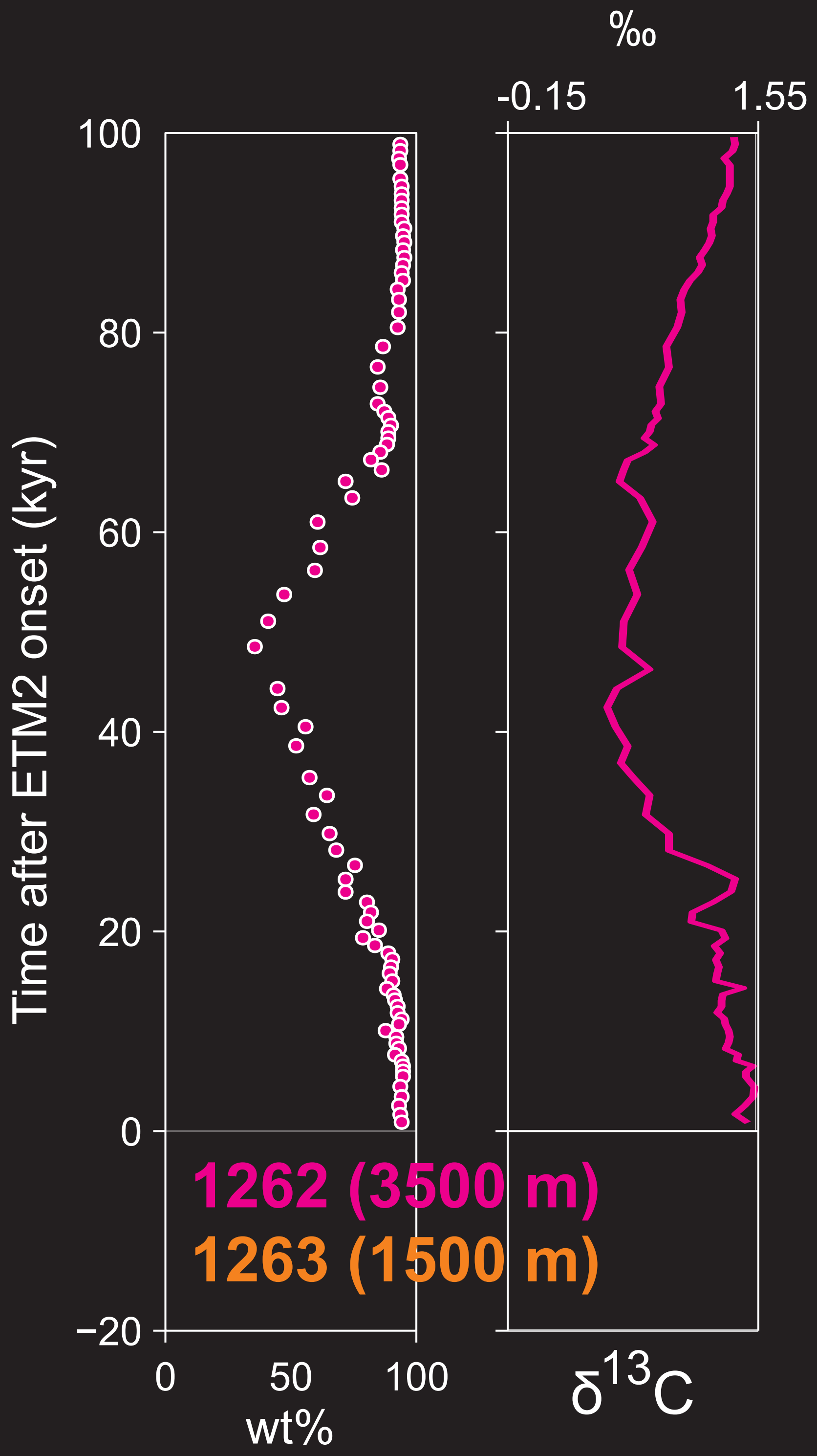

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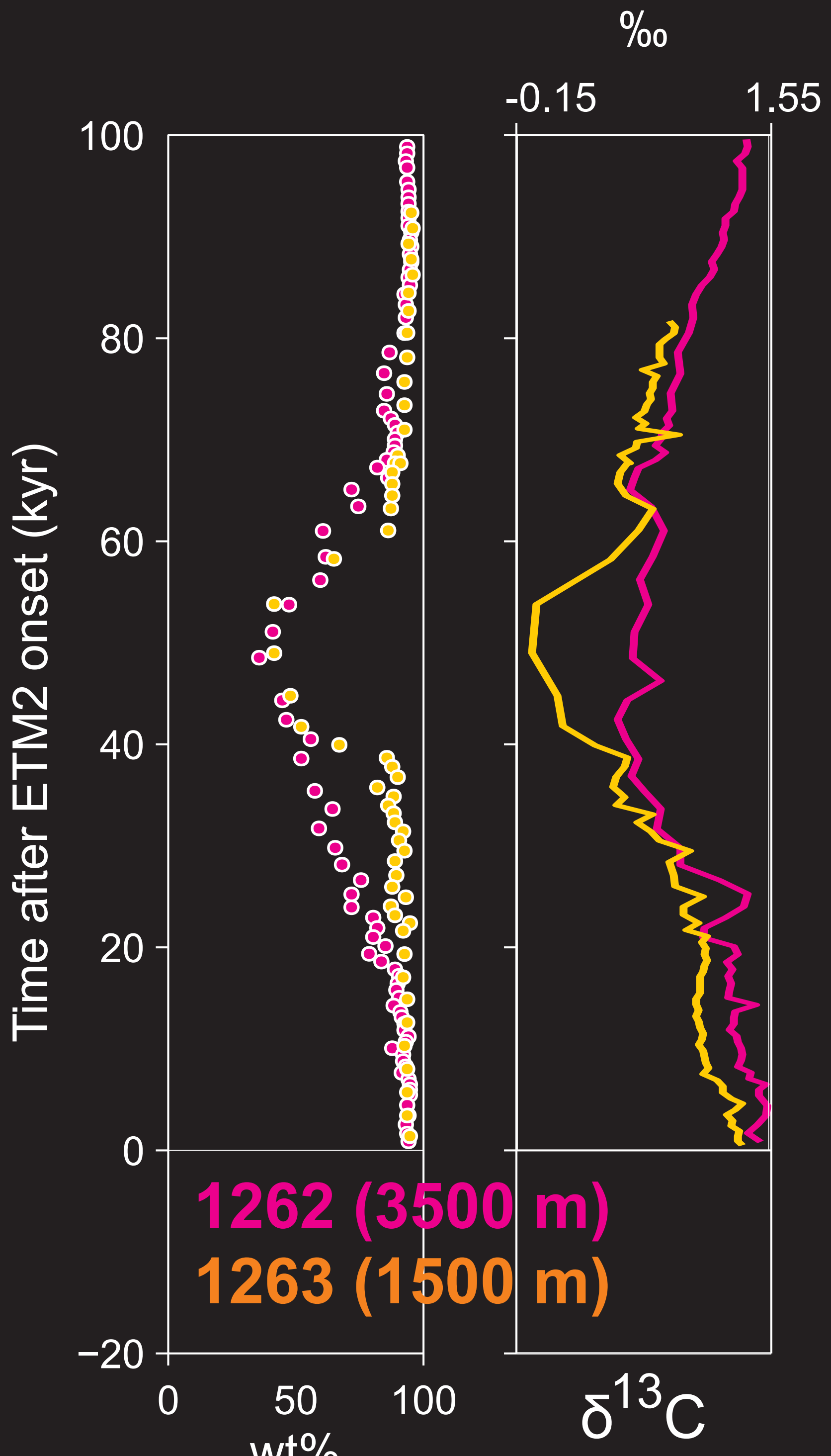
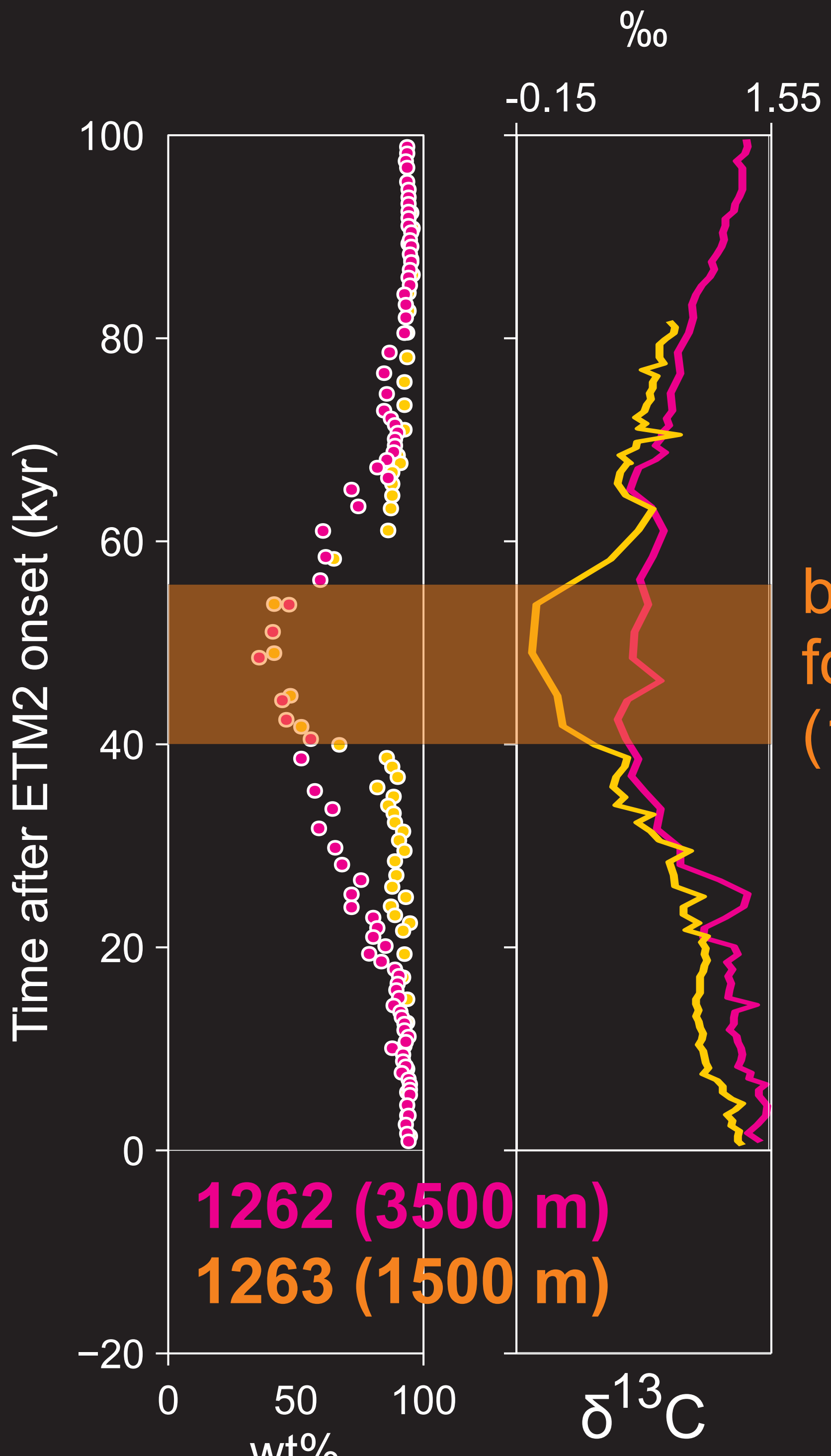
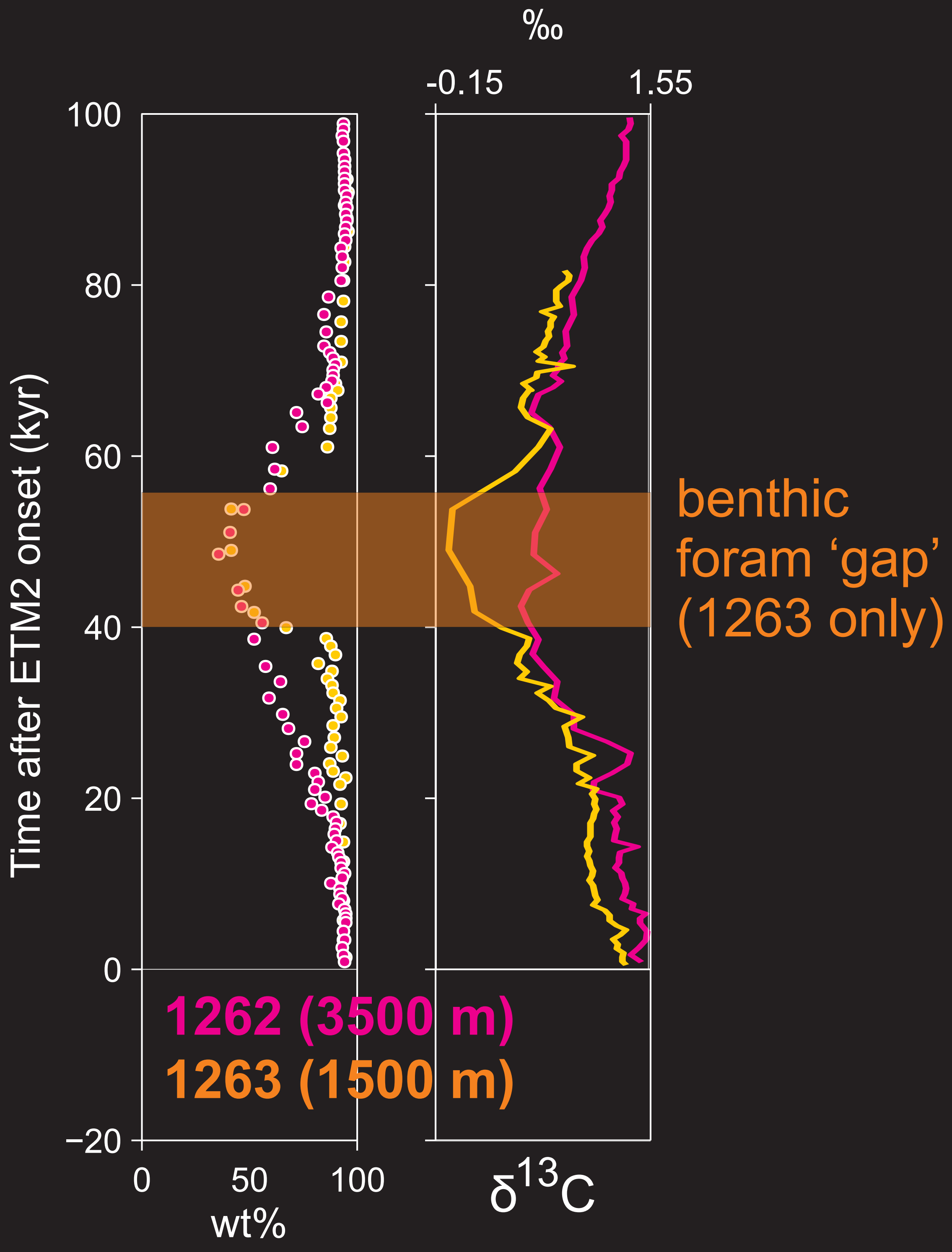


Fun with models and data: ETM2



benthic
foram 'gap'
(1263 only)

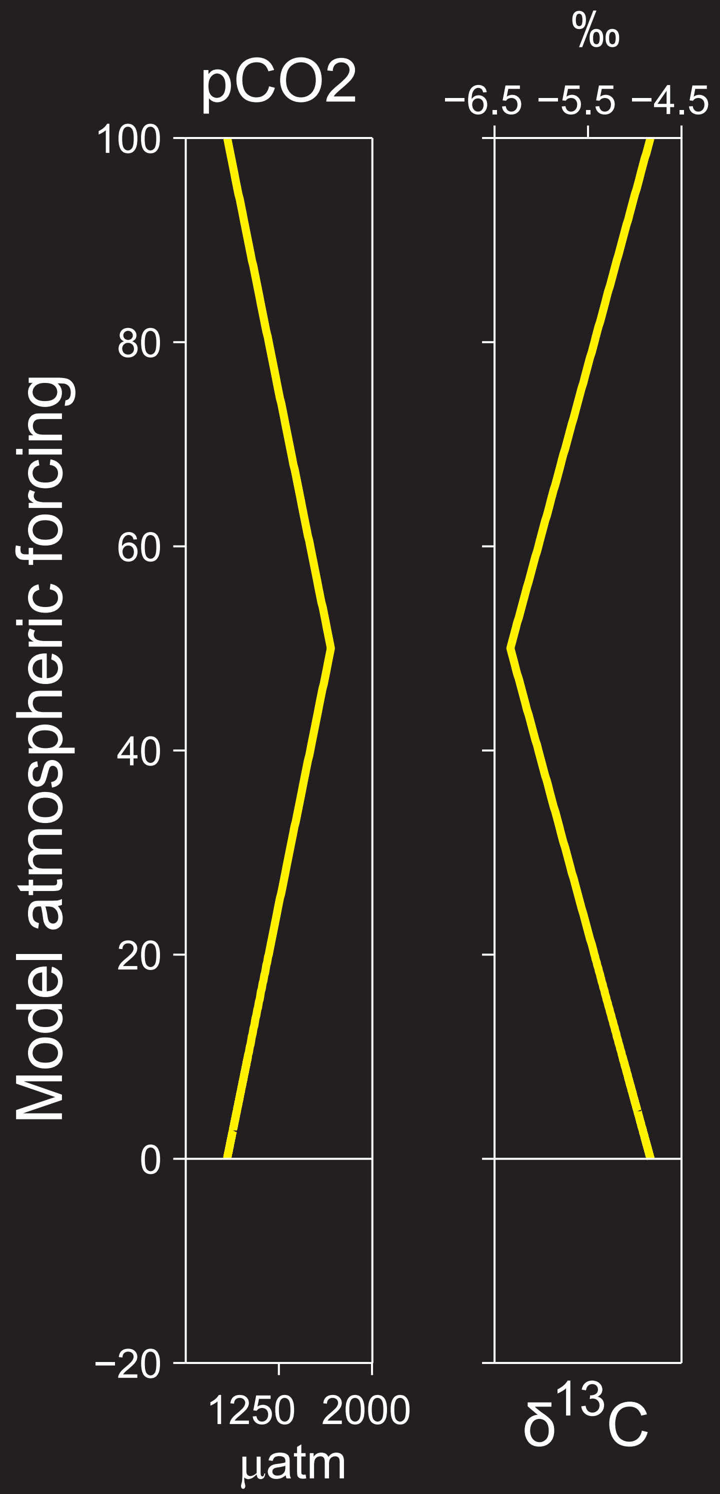
Fun with models and data: ETM2



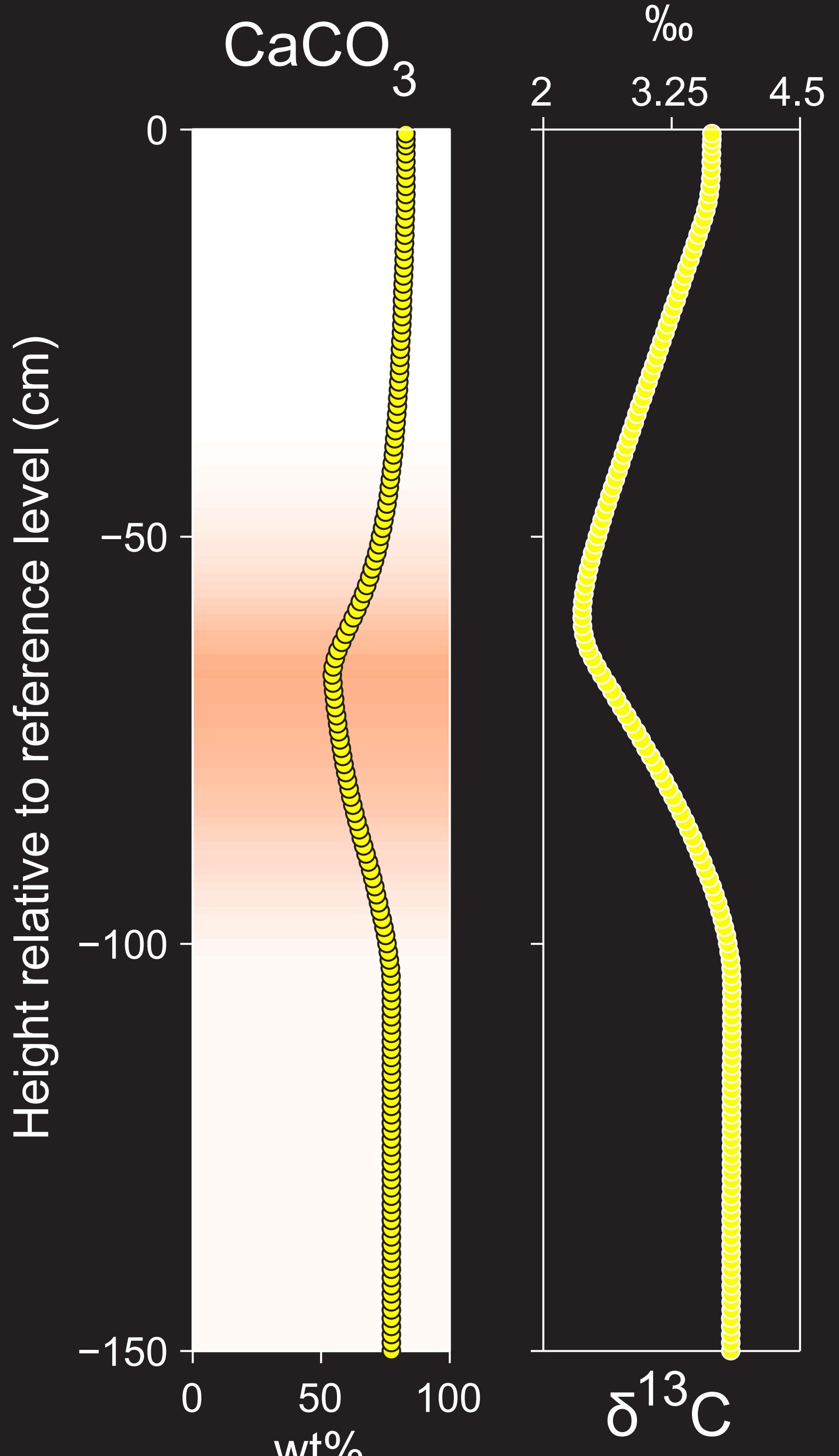
benthic foram 'gap' (1263 only)

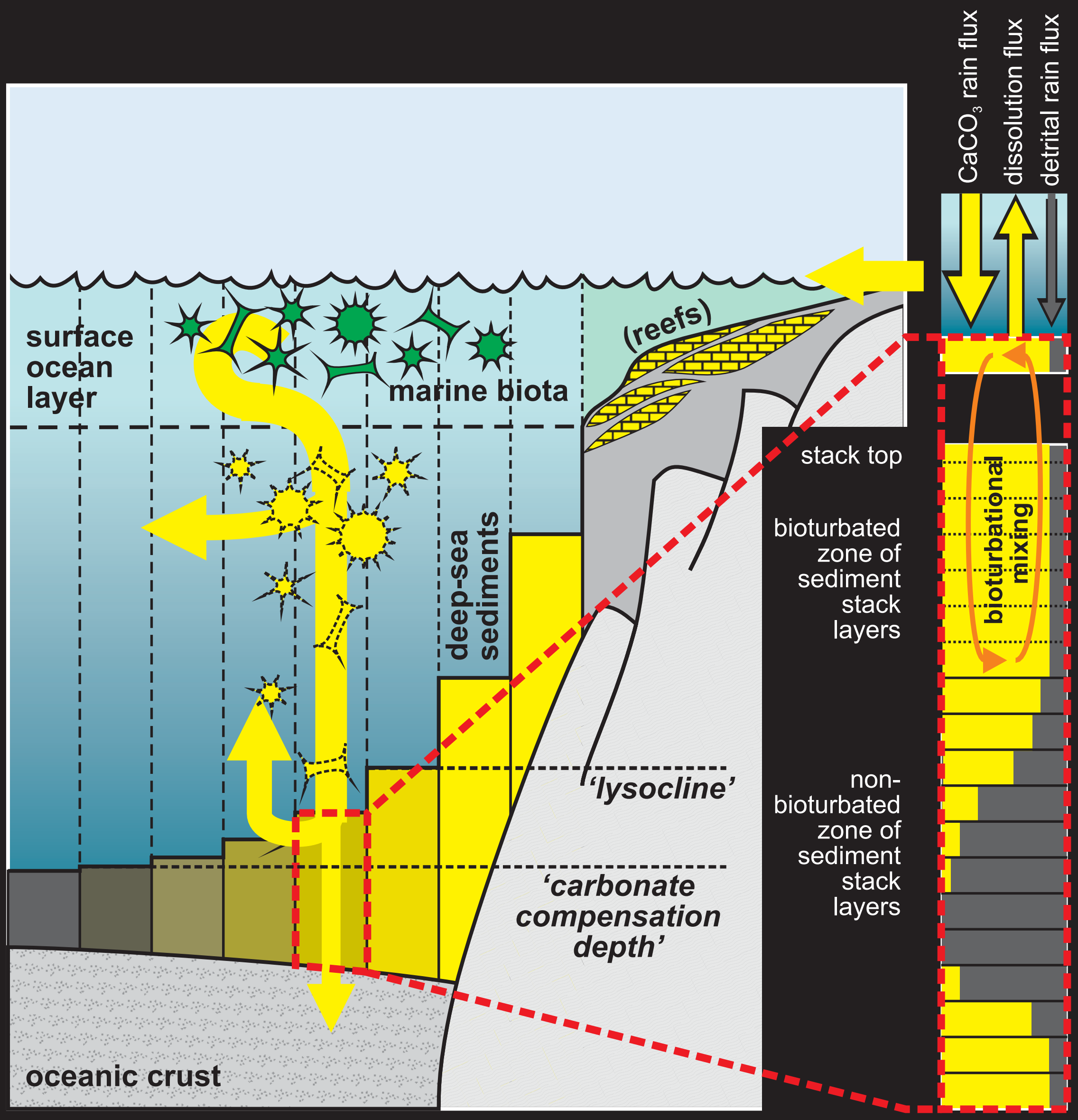
1262 (3500 m)

1263 (1500 m)

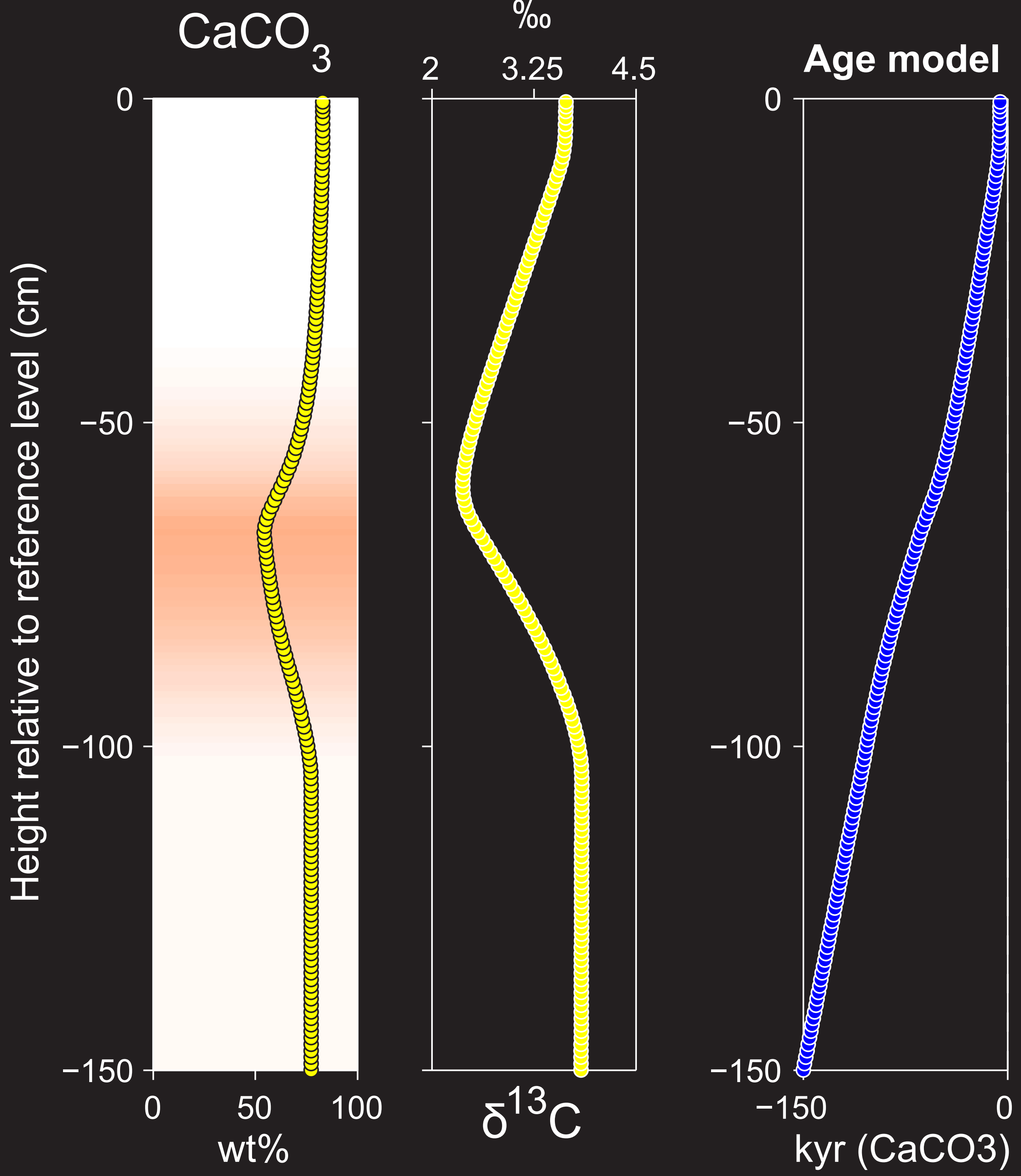


Fun with models and data: ETM2

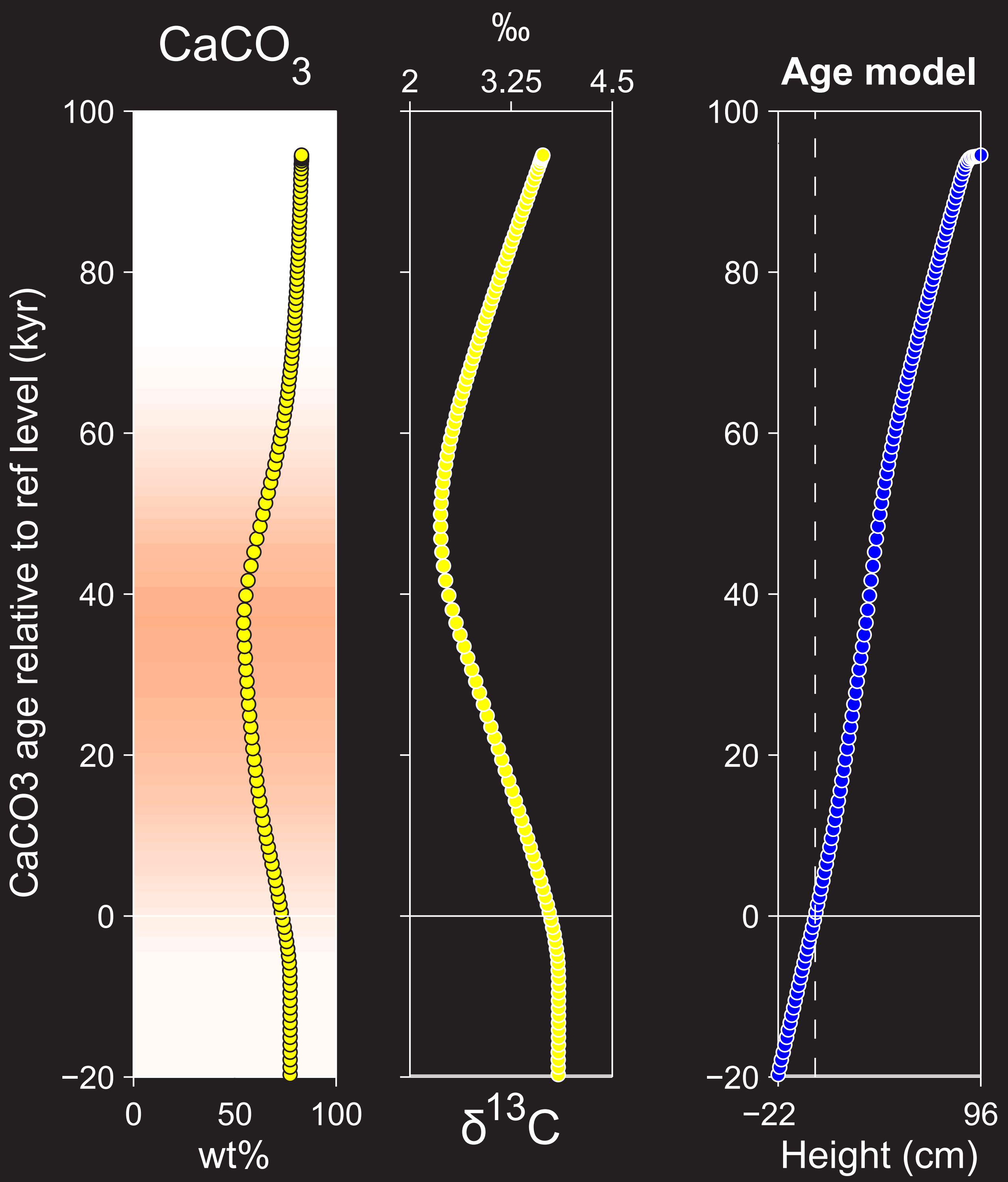




Fun with models and data: ETM2



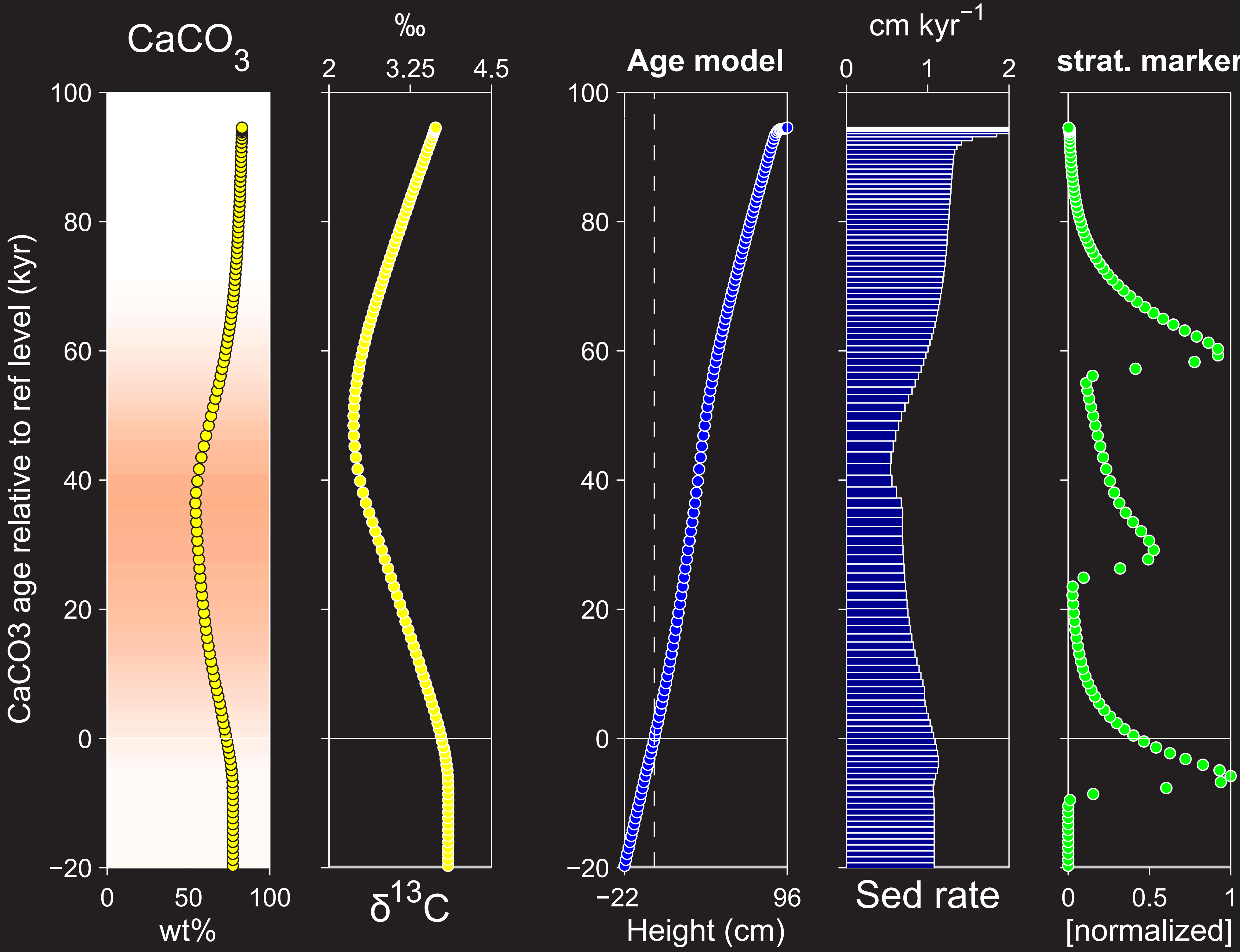
Fun with models and data: ETM2



The 'perfect' (age) model ...

... with no connection with 'reality' (what is measurable and knowable).

Fun with models and data: ETM2



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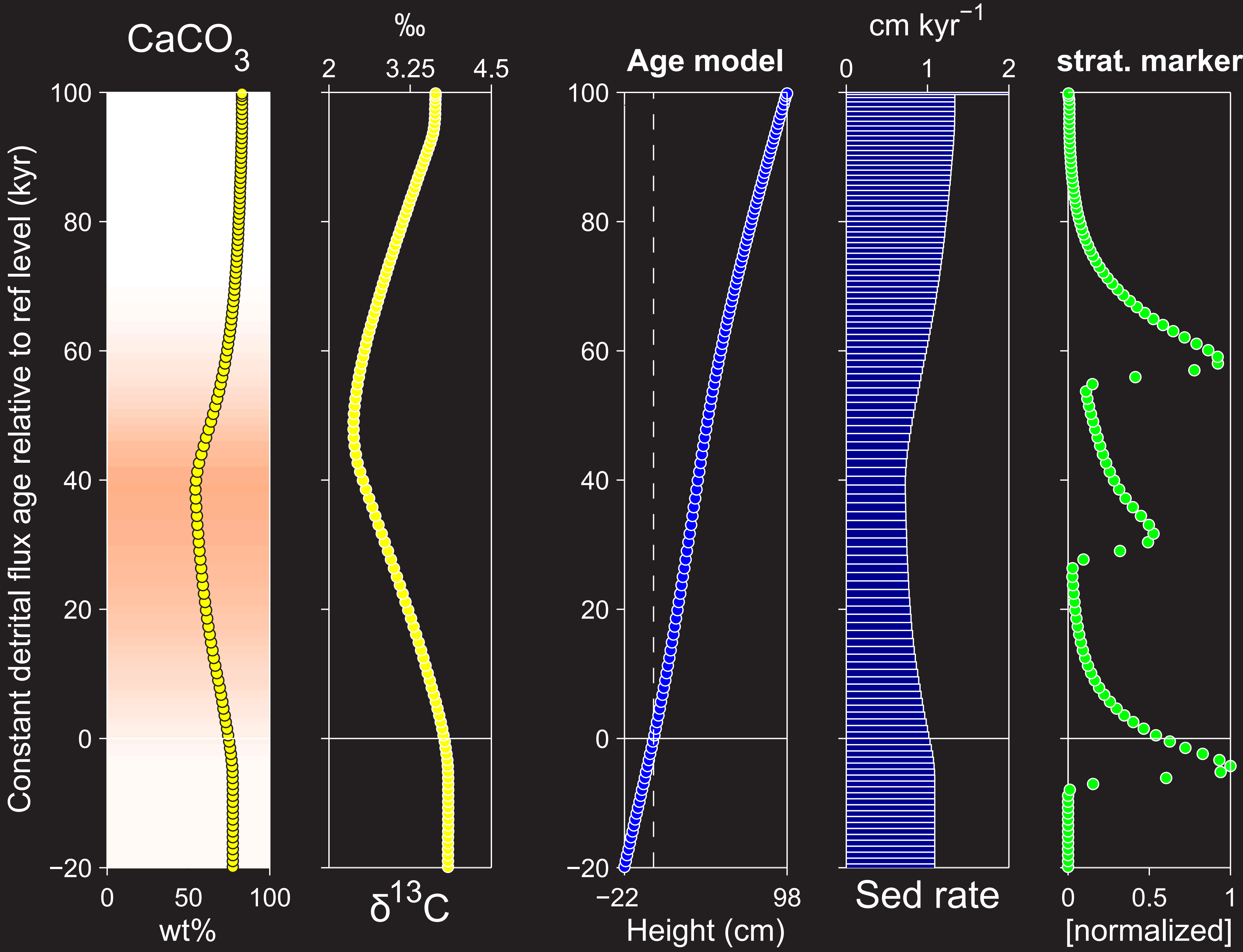
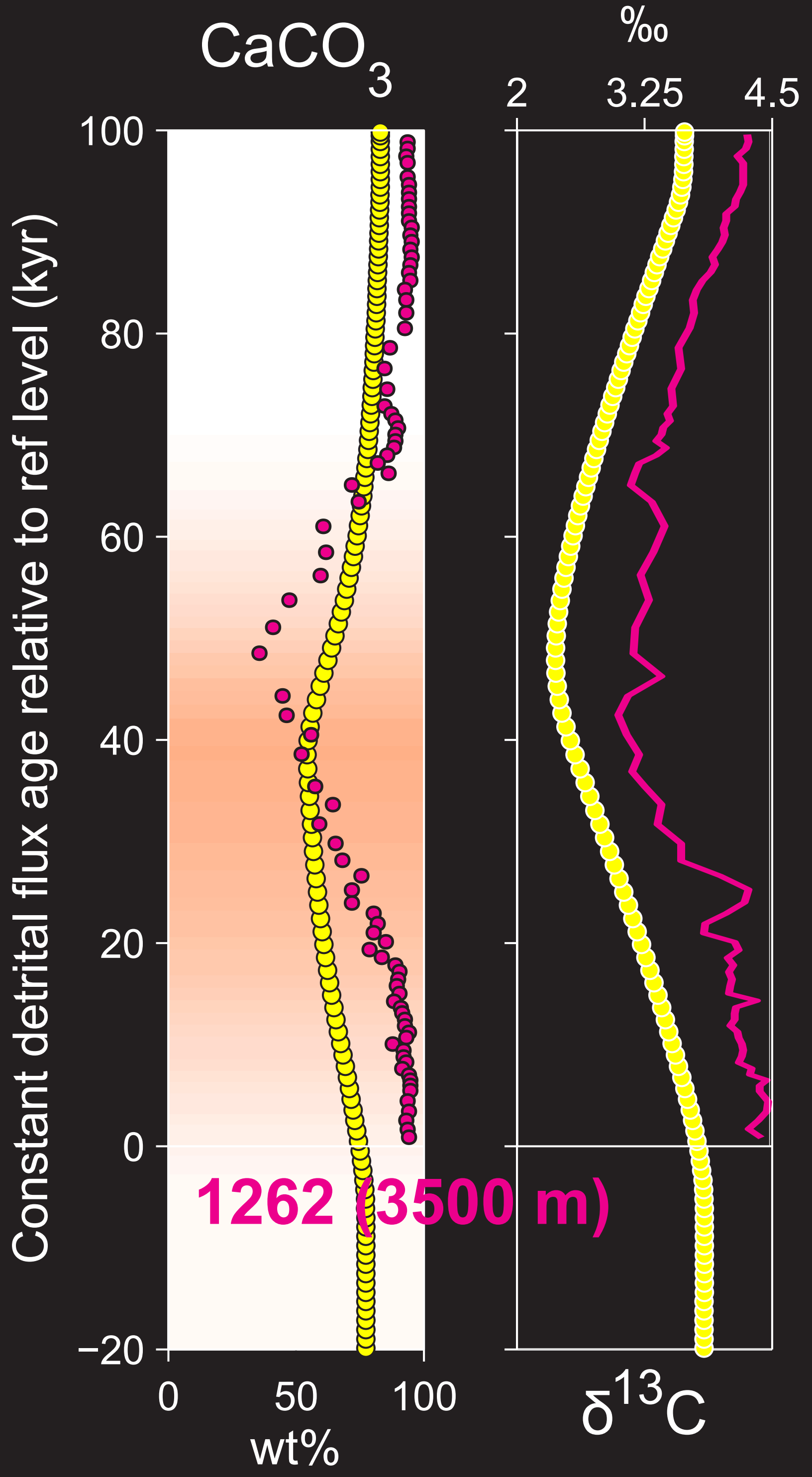


Fun with models and data: ETM2 and ‘interface’ vs. ‘homogeneous’ dissolution



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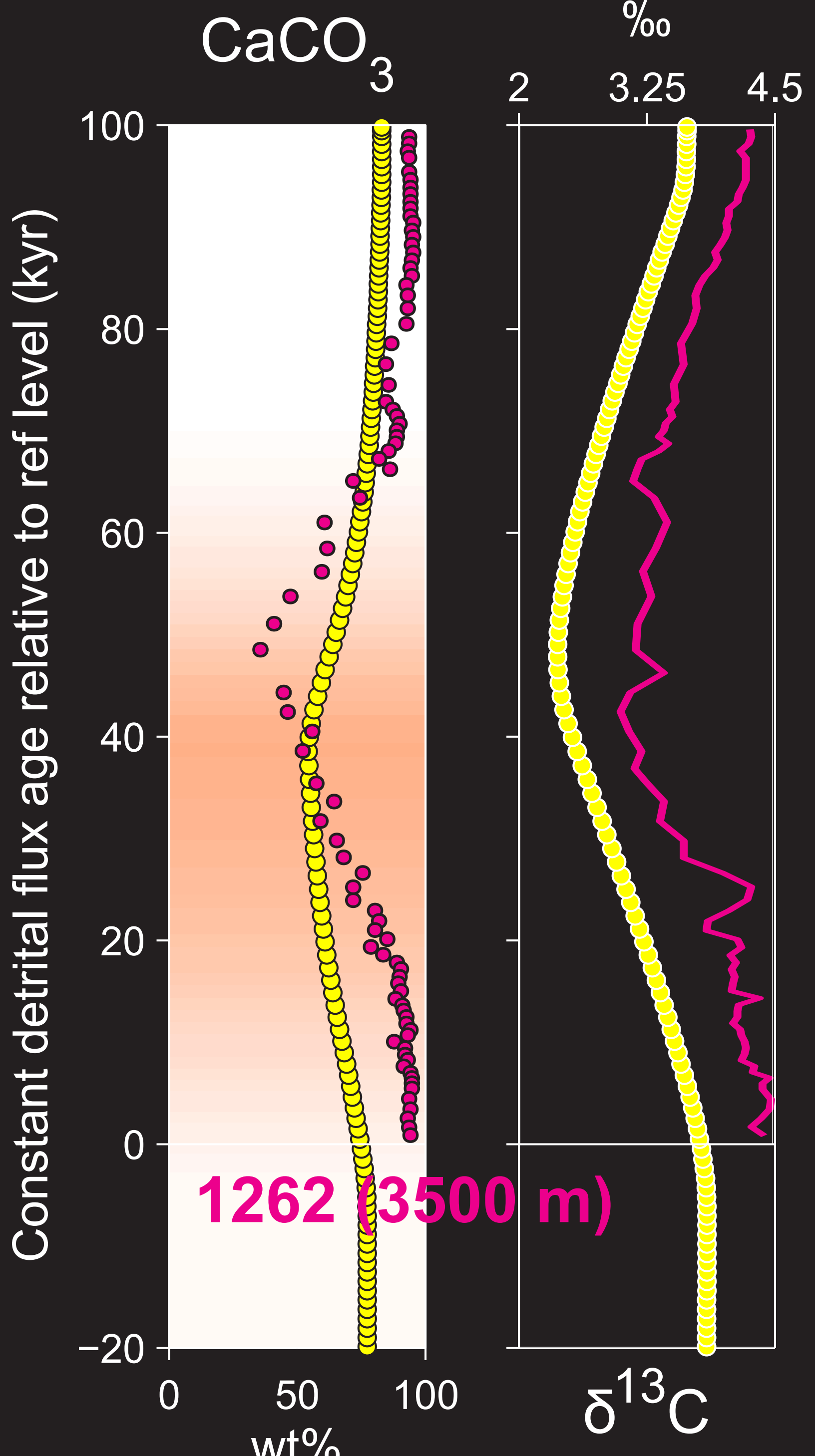

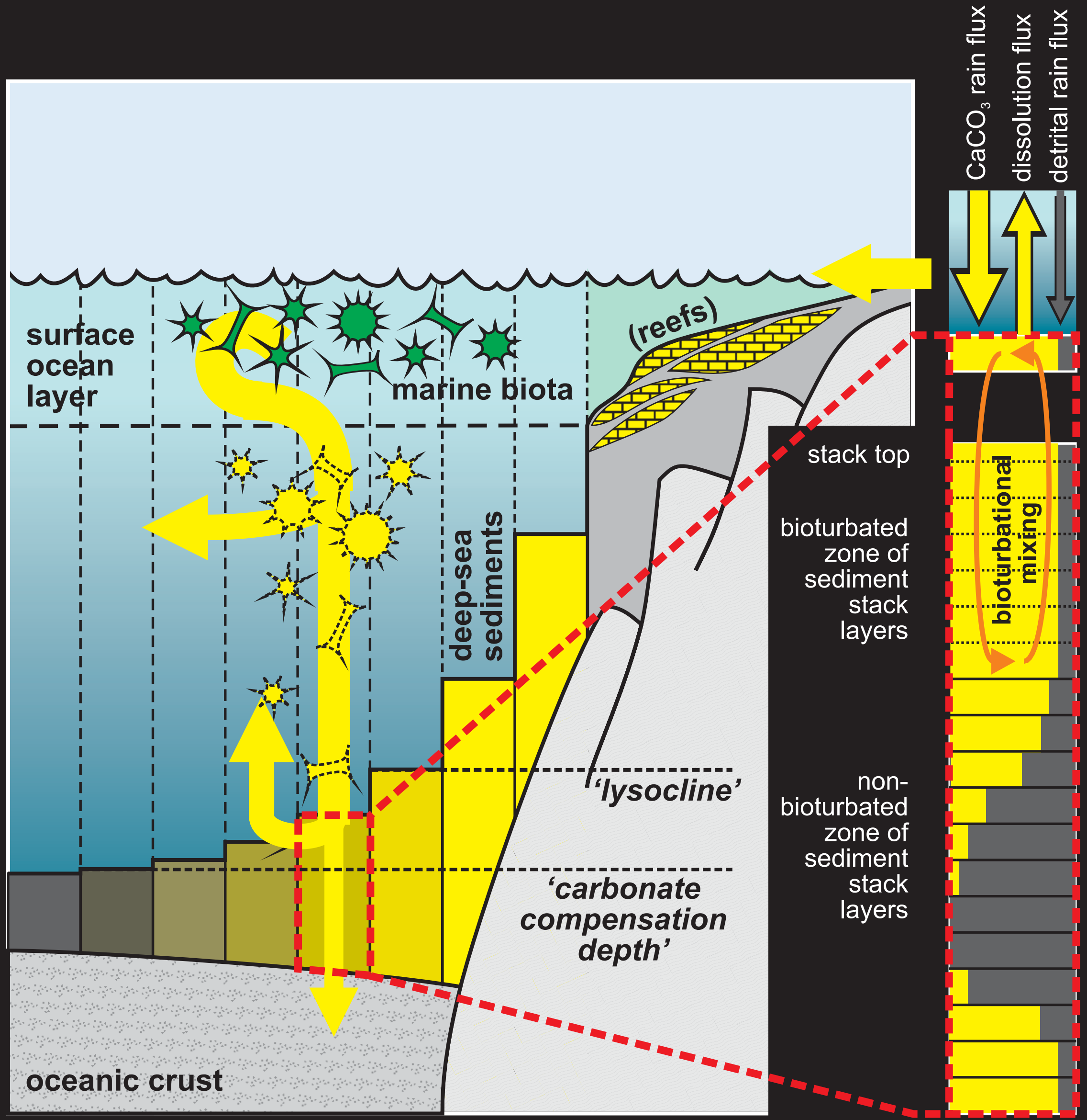

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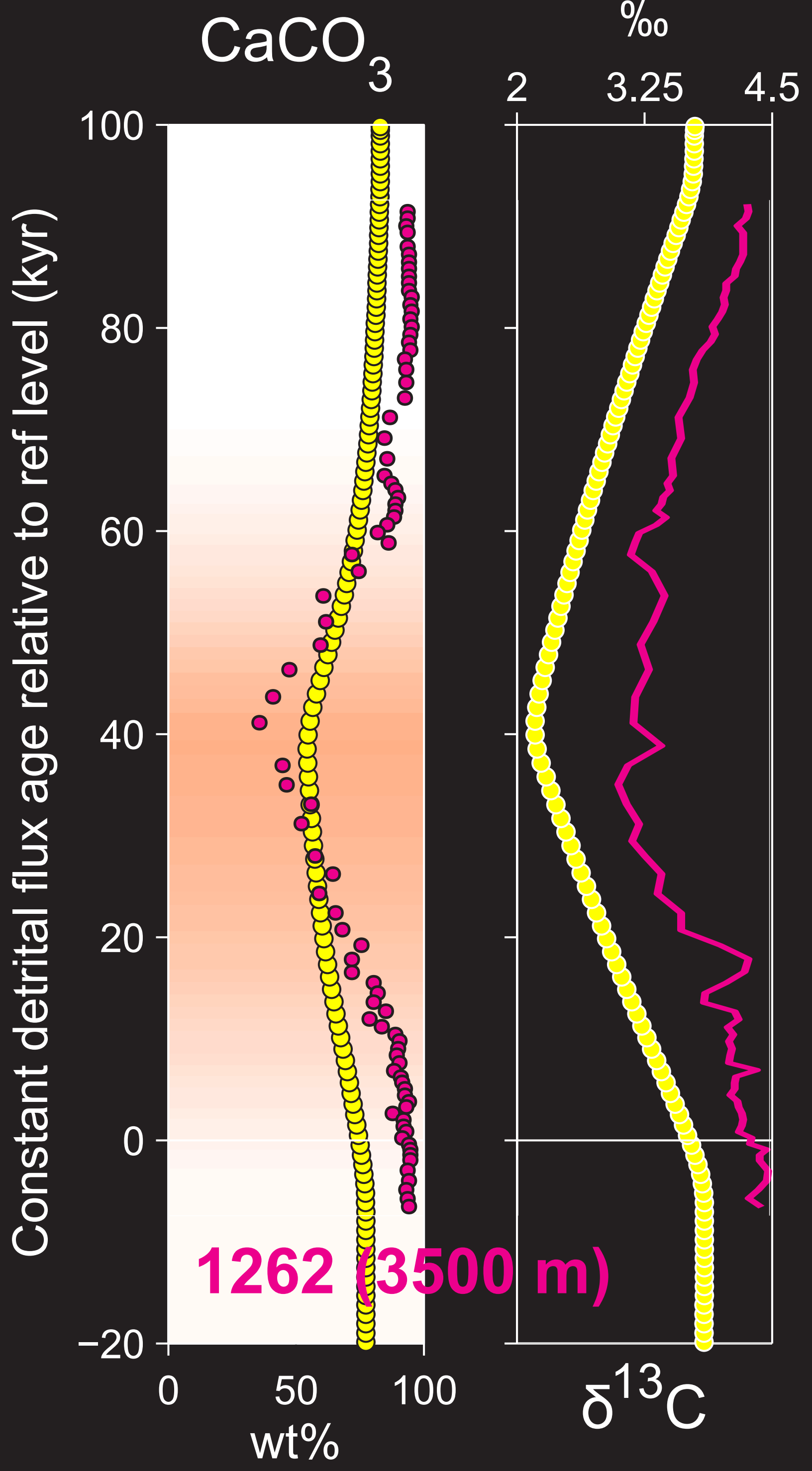
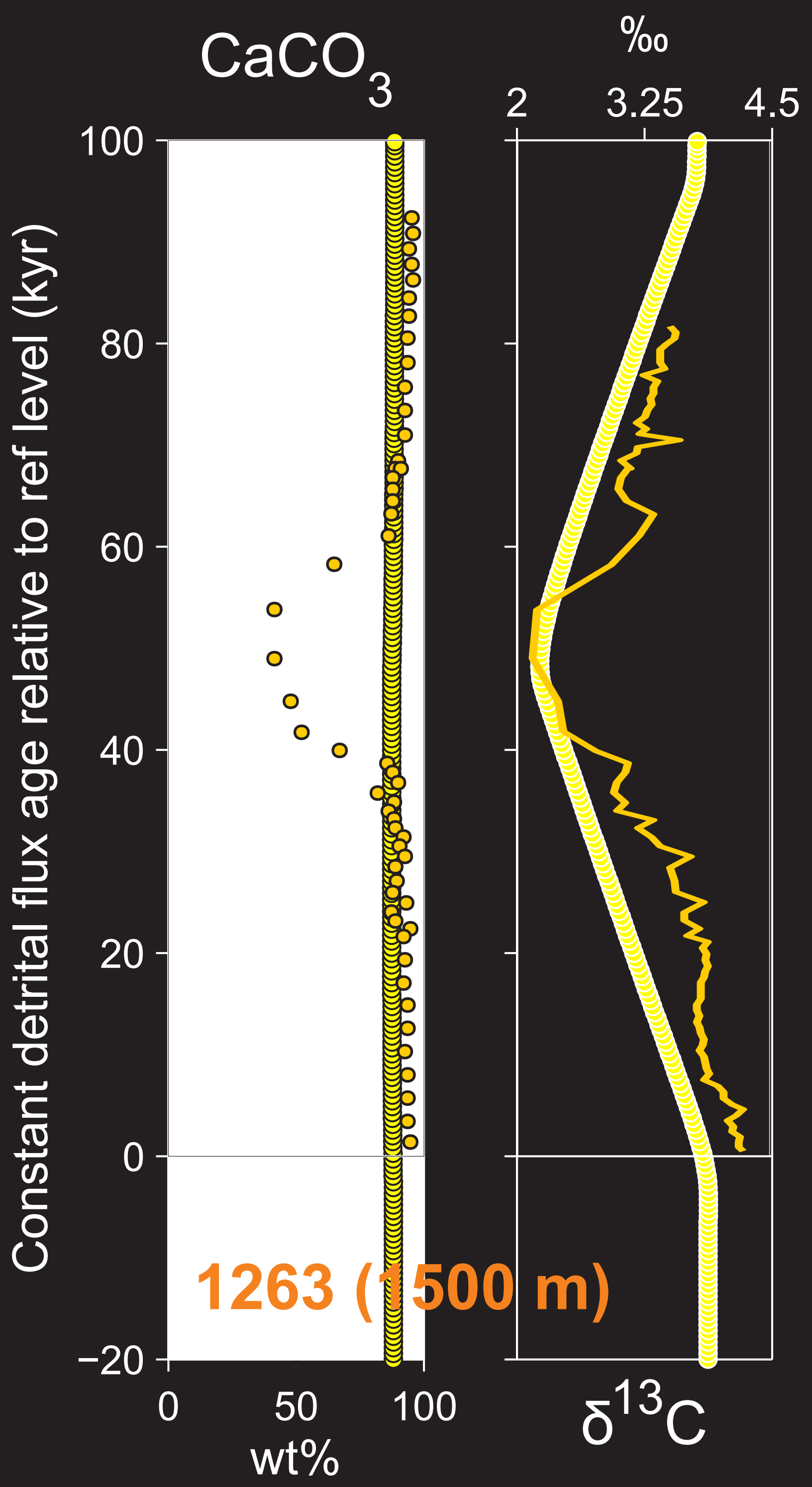
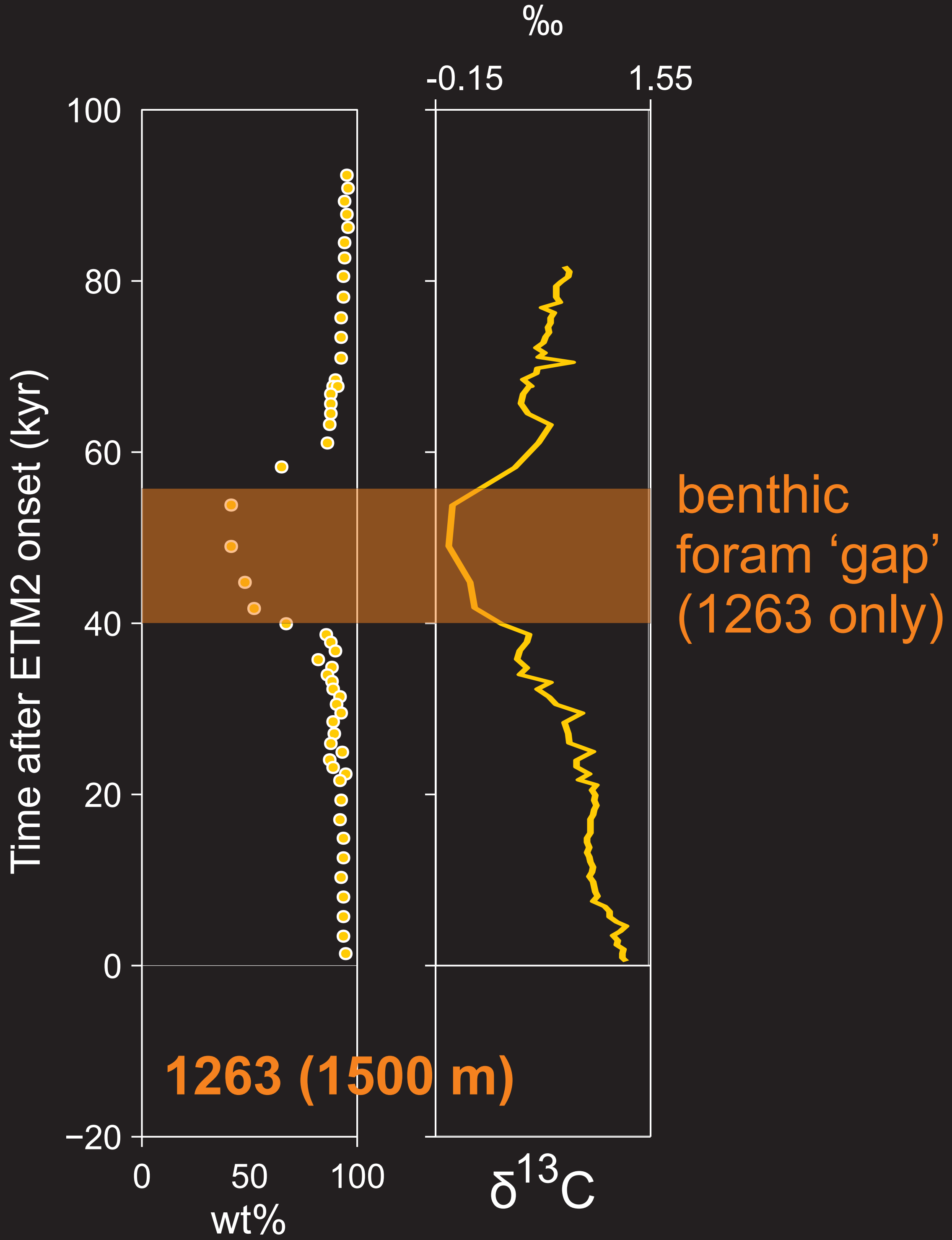


Fun with models and data: ETM2

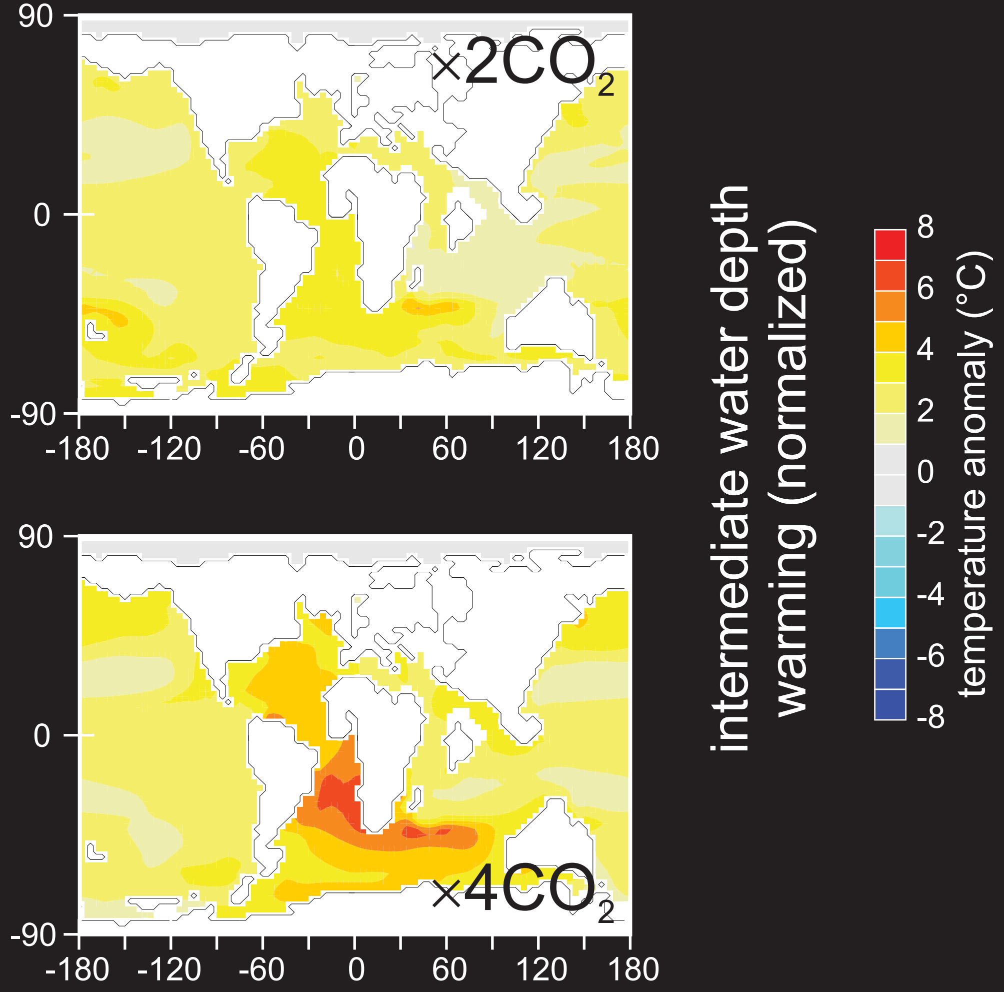


Fun with models and data: ETM2

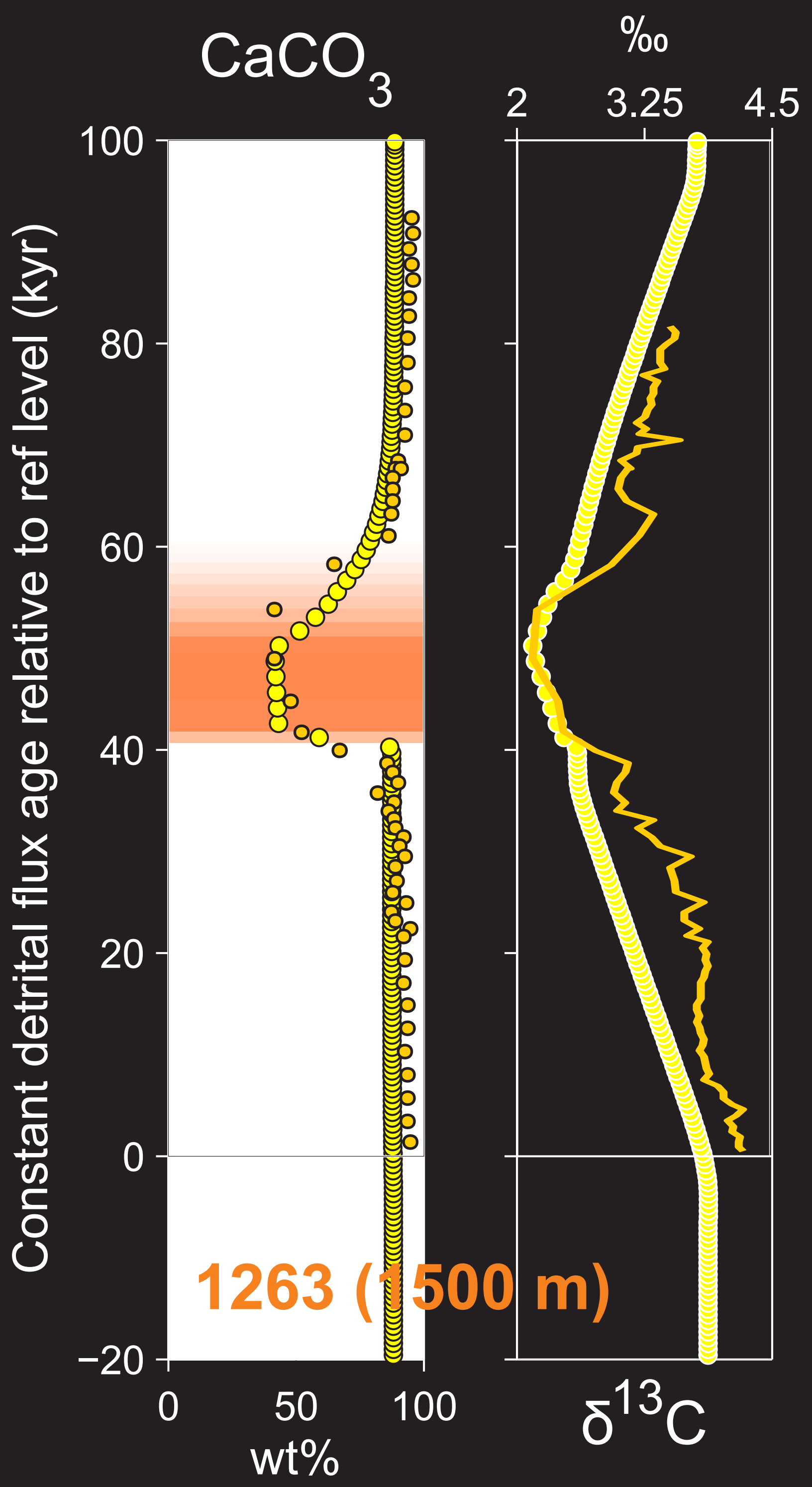
Lunt et al. [2010, 2011]



benthic foram 'gap' (1263 only)

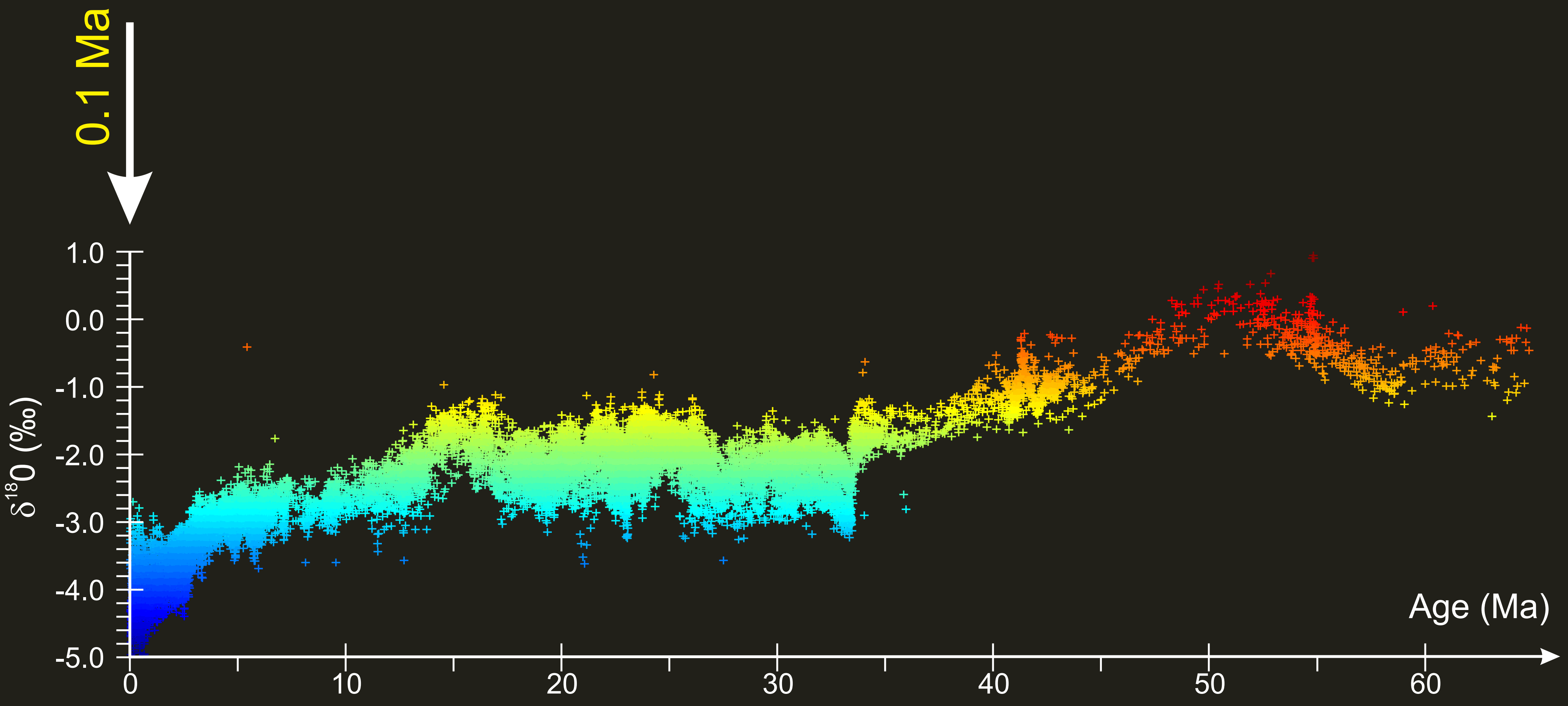


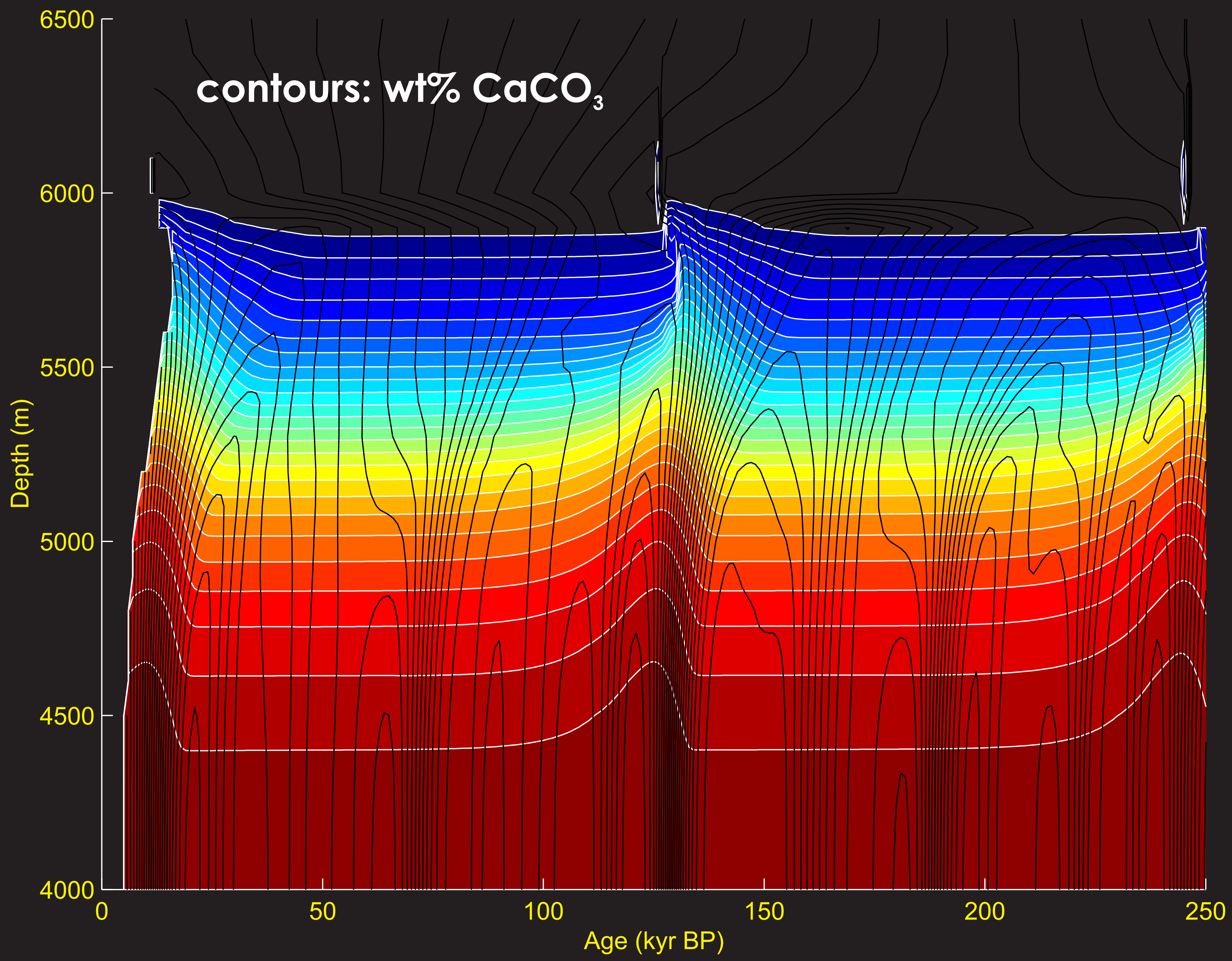
Fun with models and data: ETM2

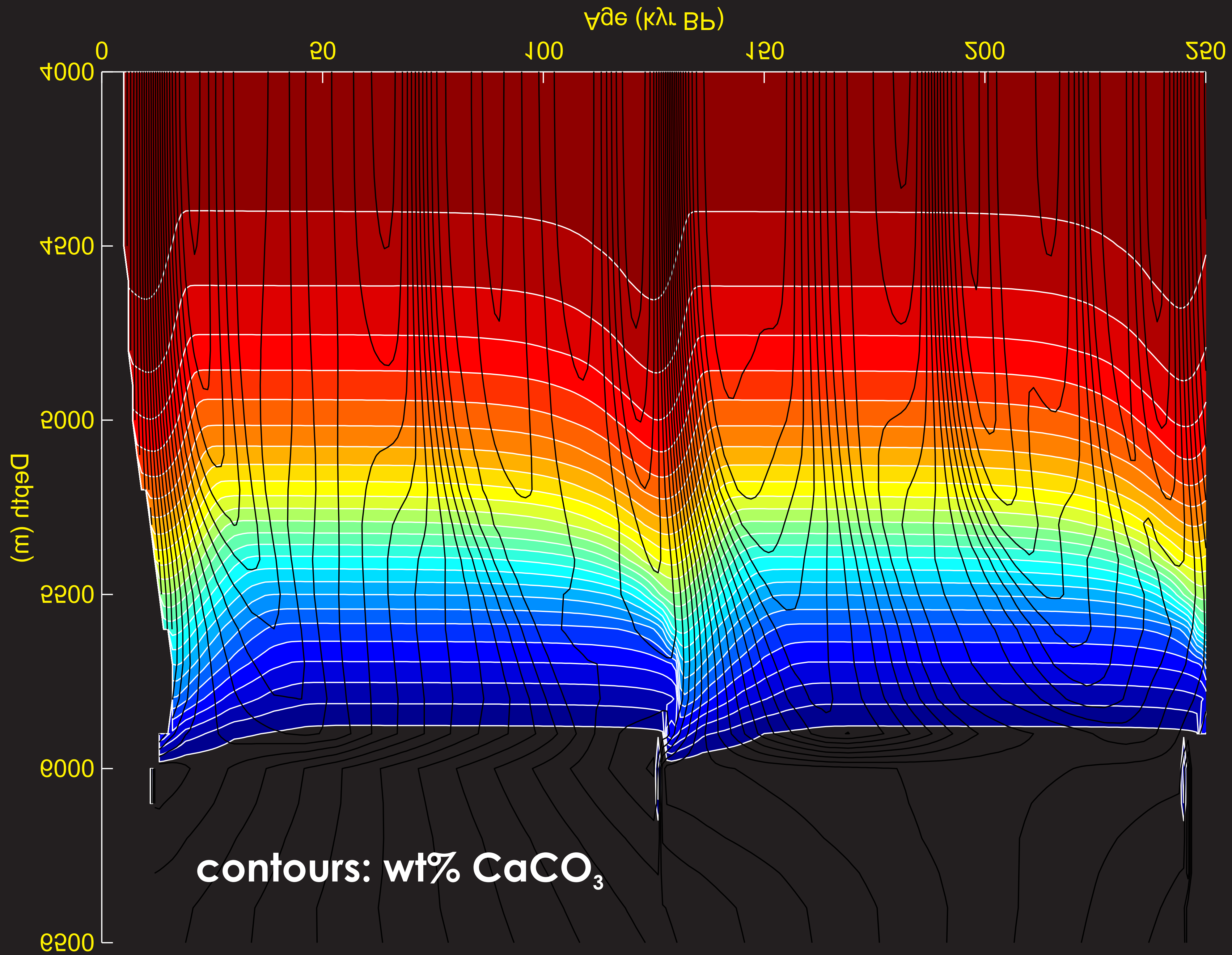


Consider:
Co-varying (glacial-interglacial) $CaCO_3$ dissolution cycles and how a (varying) stable isotope is recorded

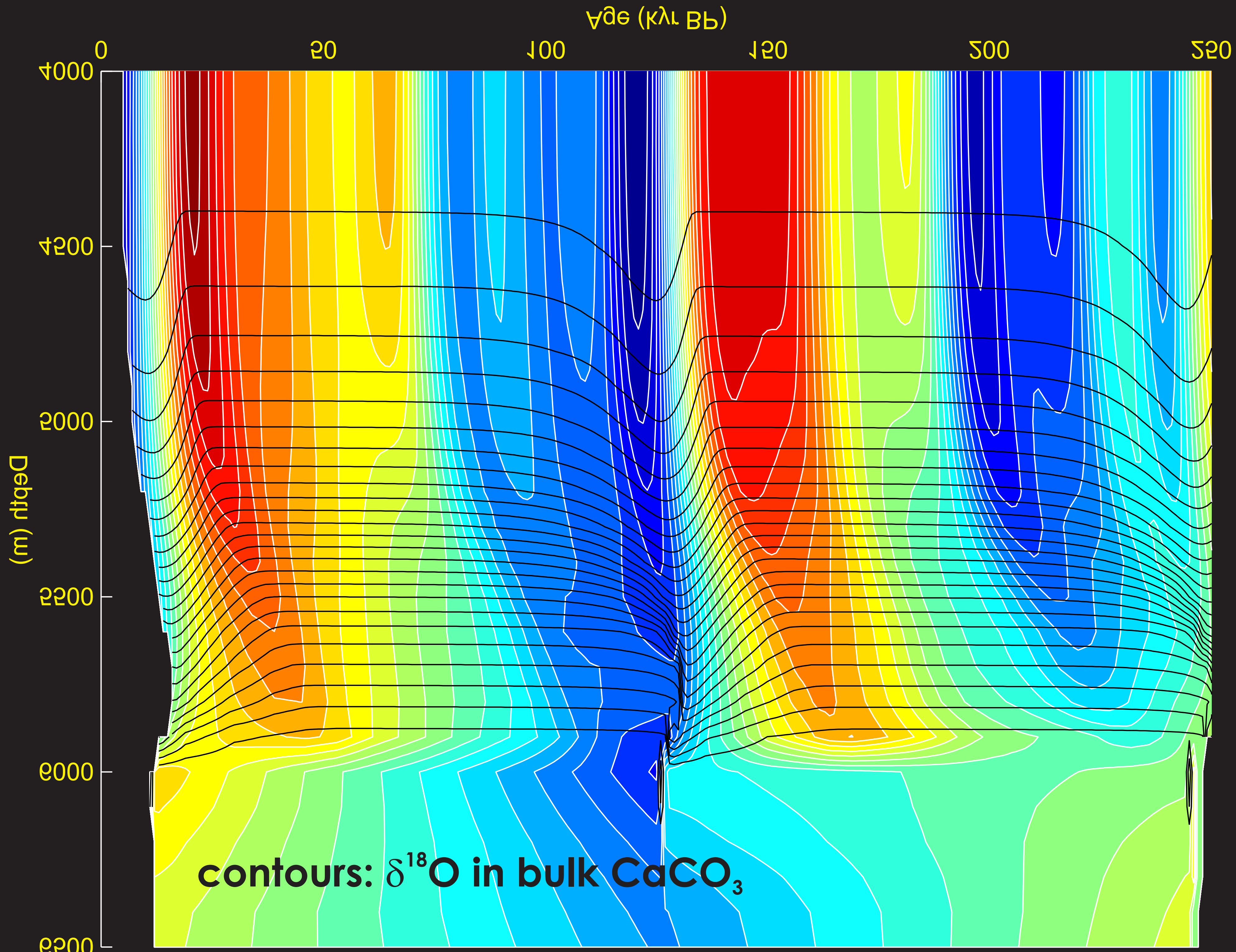
[Here: $\delta^{18}O$ of planktic carbonate following SPECMAP plus ...
500 PgC CO_2 removed from the atmosphere (to the terrestrial biosphere) across the deglacial transition (and then gradually added back again).]




contours: wt% $CaCO_3$
6500
6000
5500
5000
4500
4000
Depth (m)
0
50
100
150
200
250
Age (kyr BP)



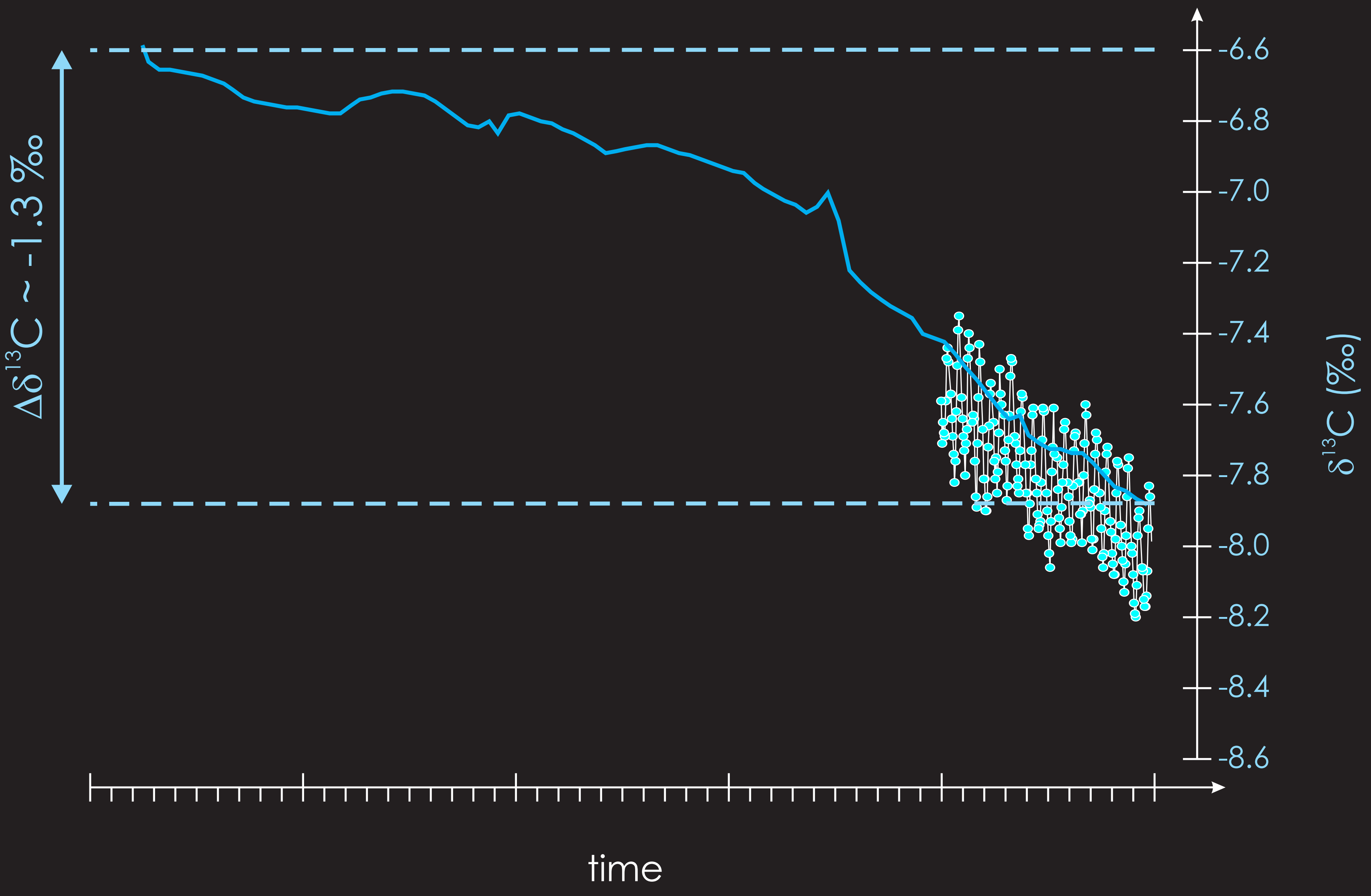
contours: wt% $CaCO_3$



contours: $\delta^{18}O$ in bulk $CaCO_3$

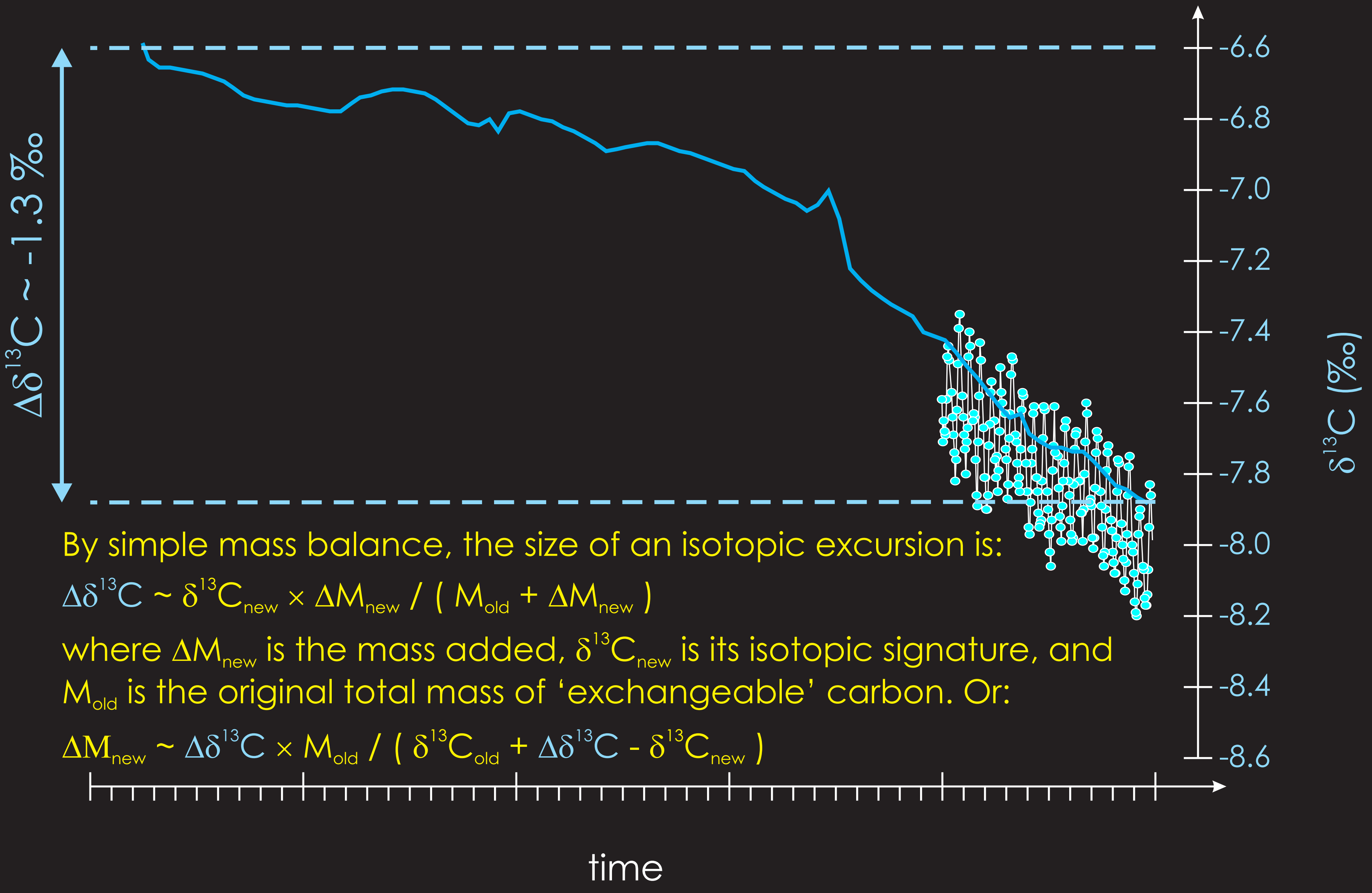
Fun with models and data:

Quantifying carbon release



Fun with models and data:

Quantifying carbon release – mass balance estimate approach



By simple mass balance, the size of an isotopic excursion is:

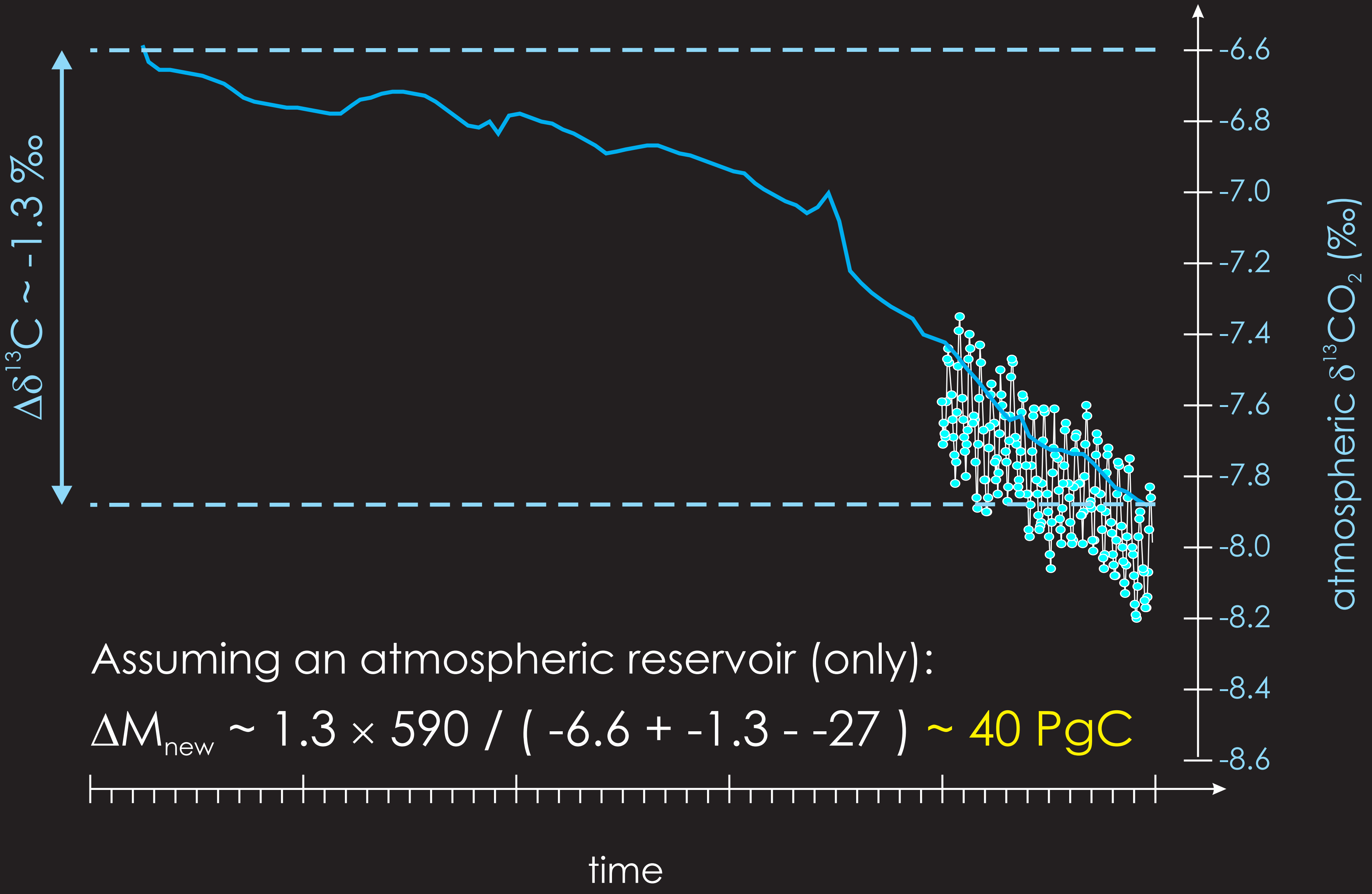
$$\Delta\delta^{13}C \sim \delta^{13}C_{new} \times \Delta M_{new} / (M_{old} + \Delta M_{new})$$

where ΔM_{new} is the mass added, $\delta^{13}C_{new}$ is its isotopic signature, and M_{old} is the original total mass of ‘exchangeable’ carbon. Or:

$$\Delta M_{new} \sim \Delta\delta^{13}C \times M_{old} / (\delta^{13}C_{old} + \Delta\delta^{13}C - \delta^{13}C_{new})$$

Fun with models and data:

Quantifying carbon release – mass balance estimate approach

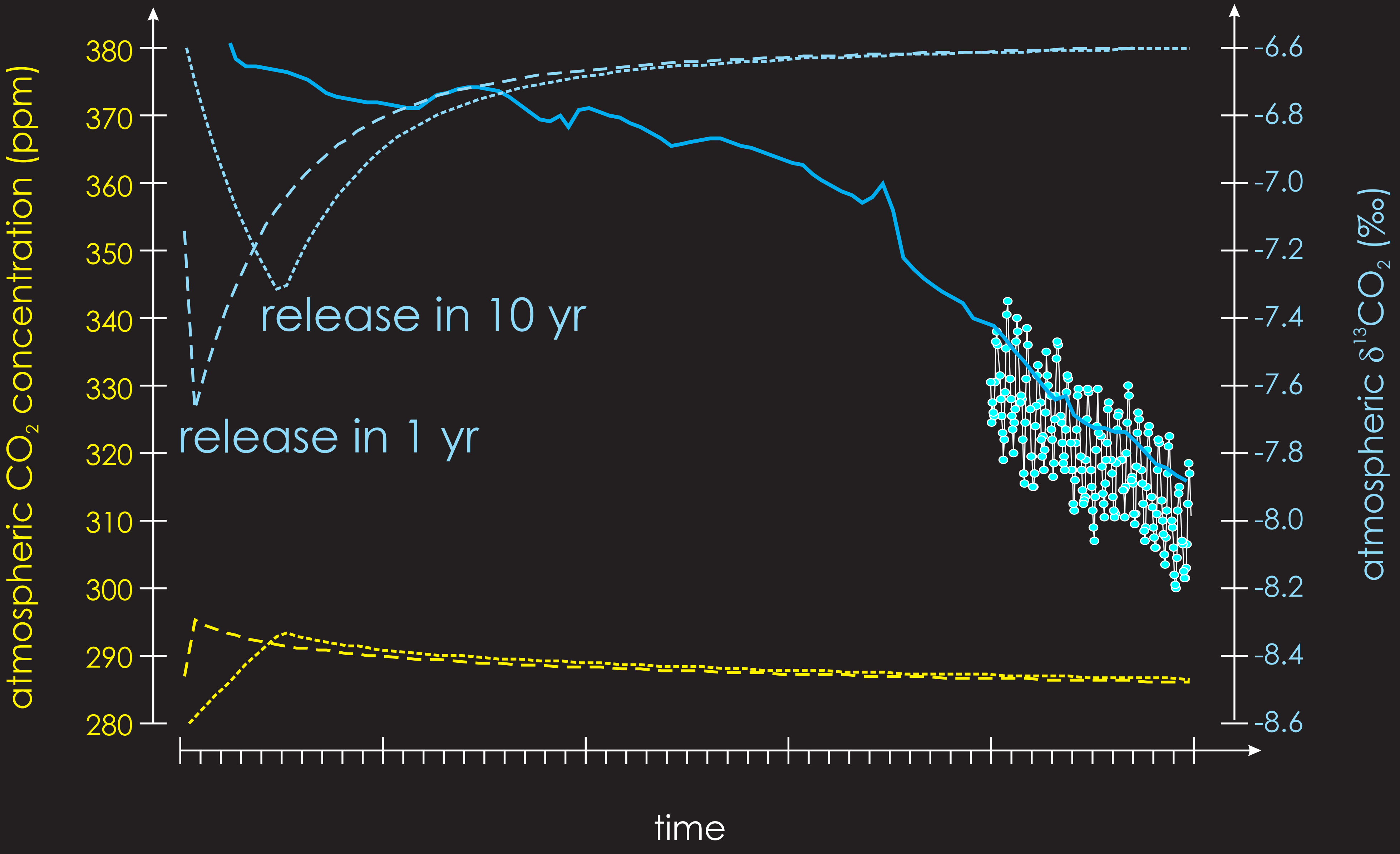


Assuming an atmospheric reservoir (only):

$$\Delta M_{new} \sim 1.3 \times 590 / (-6.6 + -1.3 - -27) \sim 40 \text{ PgC}$$

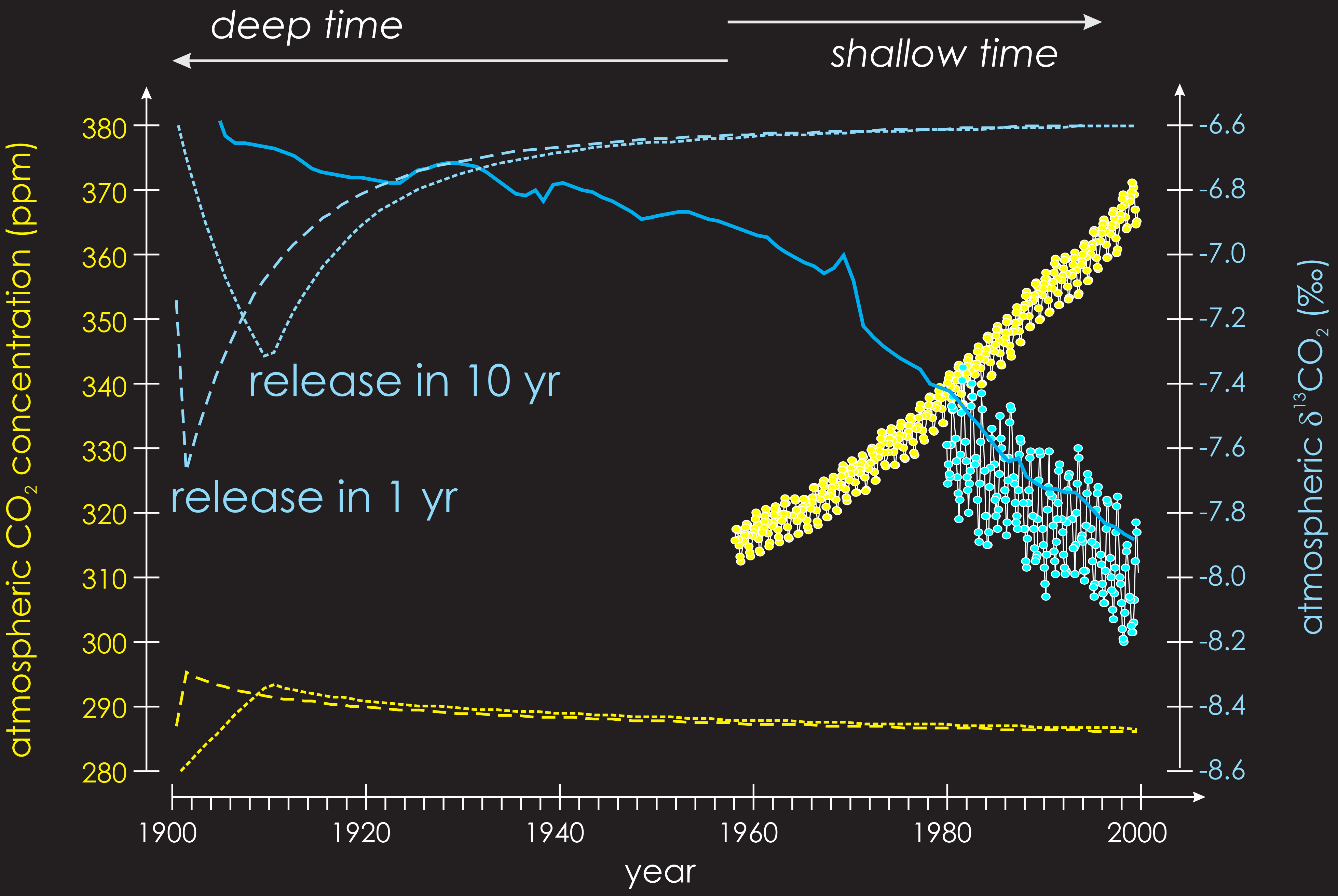
Fun with models and data:

Quantifying carbon release – model trial-and-error approach



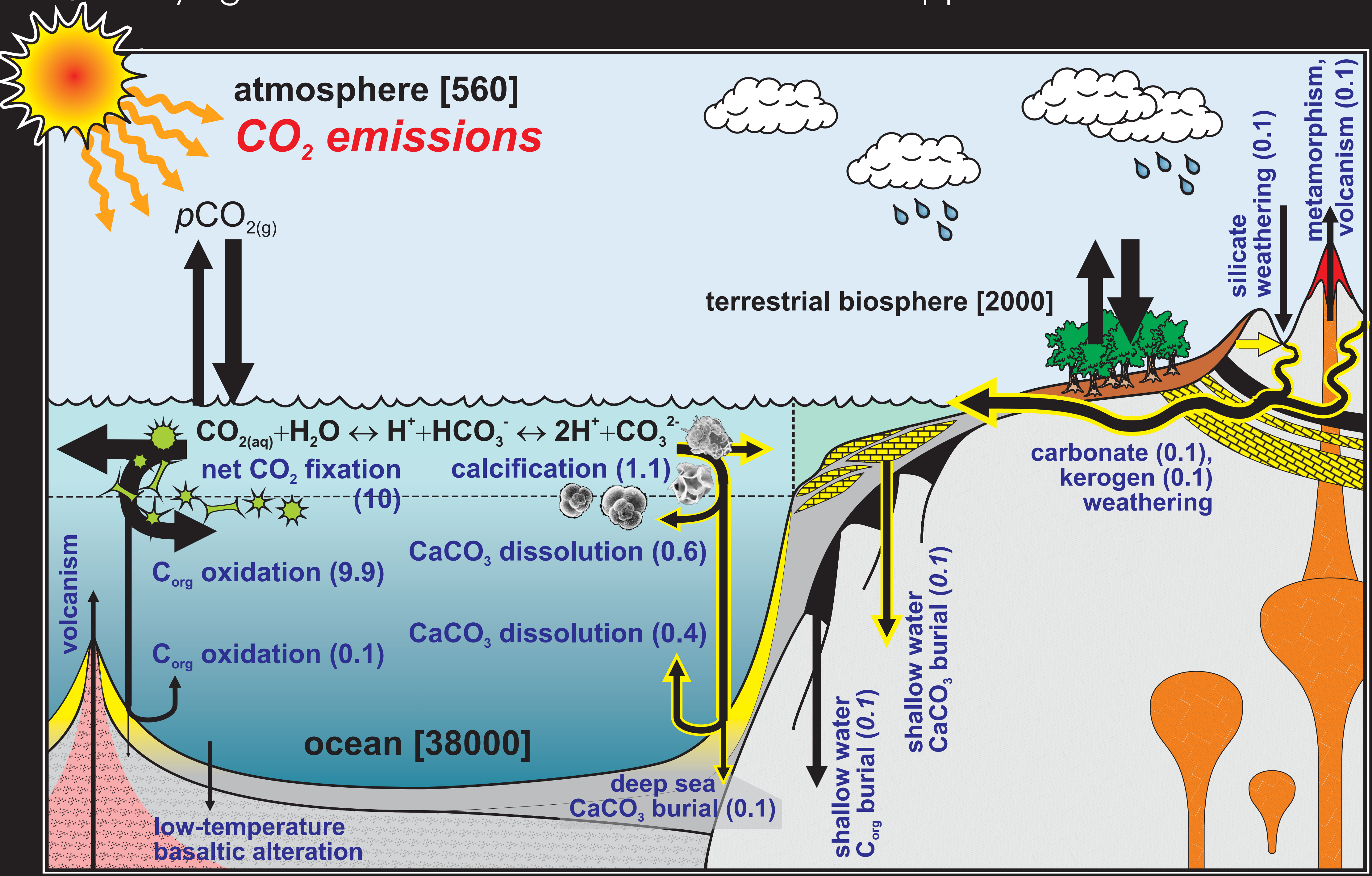
Fun with models and data:

Quantifying carbon release – model trial-and-error approach



Fun with models and data:

Quantifying carbon release – model trial-and-error approach



Fun with models and data:
Data assimilation

2 The EnKF

The EnKF is now briefly described with focus on notation and the standard analysis scheme. The notation follows that used in Evensen (2003).

2.1 Ensemble representation for P

As in Evensen (2003), we have defined the matrix holding the ensemble members $\psi_i \in \Re^n$,

$$\boldsymbol{A} = (\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots, \boldsymbol{\psi}_N) \in \Re^{n \times N}, \tag{1}$$

where N is the number of ensemble members and n is the size of the model state vector.

The ensemble mean is stored in each column of $\overline{\boldsymbol{A}}$ which can be defined as

$$\overline{\boldsymbol{A}} = \boldsymbol{A}\boldsymbol{1}_N, \tag{2}$$

where $\boldsymbol{1}_N \in \Re^{N \times N}$ is the matrix where each element is equal to $1/N$. We can then define the ensemble perturbation matrix as

$$\boldsymbol{A}' = \boldsymbol{A} - \overline{\boldsymbol{A}} = \boldsymbol{A}(\boldsymbol{I} - \boldsymbol{1}_N). \tag{3}$$

The ensemble covariance matrix $\boldsymbol{P}_\mathrm{e} \in \Re^{n \times n}$ can be defined as

$$\boldsymbol{P}_\mathrm{e} = \frac{\boldsymbol{A}'(\boldsymbol{A}')^\mathrm{T}}{N-1}. \tag{4}$$

2.2 Measurement perturbations

Given a vector of measurements $\boldsymbol{d} \in \Re^m$, with m being the number of measurements, we can define the N vectors of perturbed observations as

$$\boldsymbol{d}_j = \boldsymbol{d} + \boldsymbol{\epsilon}_j, \quad j = 1, \ldots, N, \tag{5}$$

which can be stored in the columns of a matrix

$$\boldsymbol{D} = (\boldsymbol{d}_1, \boldsymbol{d}_2, \ldots, \boldsymbol{d}_N) \in \Re^{m \times N}, \tag{6}$$

while the ensemble of perturbations, with ensemble mean equal to zero, can be stored in the matrix

$$\boldsymbol{E} = (\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \ldots, \boldsymbol{\epsilon}_N) \in \Re^{m \times N}, \tag{7}$$

from which we can construct the ensemble representation of the measurement error covariance matrix

$$\boldsymbol{R}_\mathrm{e} = \frac{\boldsymbol{E}\boldsymbol{E}^\mathrm{T}}{N-1}. \tag{8}$$

2.3 Analysis equation

The analysis equation, expressed in terms of the ensemble covariance matrices, is

$$\boldsymbol{A}^\mathrm{a} = \boldsymbol{A} + \boldsymbol{P}_\mathrm{e}\boldsymbol{H}^\mathrm{T}(\boldsymbol{H}\boldsymbol{P}_\mathrm{e}\boldsymbol{H}^\mathrm{T} + \boldsymbol{R}_\mathrm{e})^{-1}(\boldsymbol{D} - \boldsymbol{H}\boldsymbol{A}). \tag{9}$$

Using the ensemble of innovation vectors defined as

$$\boldsymbol{D}' = \boldsymbol{D} - \boldsymbol{H}\boldsymbol{A} \tag{10}$$

and the definitions of the ensemble error covariance matrices in Eqs. (4) and (8) the analysis can be expressed as

$$\boldsymbol{A}^\mathrm{a} = \boldsymbol{A} + \boldsymbol{A}'\boldsymbol{A}'^\mathrm{T}\boldsymbol{H}^\mathrm{T}\left(\boldsymbol{H}\boldsymbol{A}'\boldsymbol{A}'^\mathrm{T}\boldsymbol{H}^\mathrm{T} + \boldsymbol{E}\boldsymbol{E}^\mathrm{T}\right)^{-1}\boldsymbol{D}'. \tag{11}$$

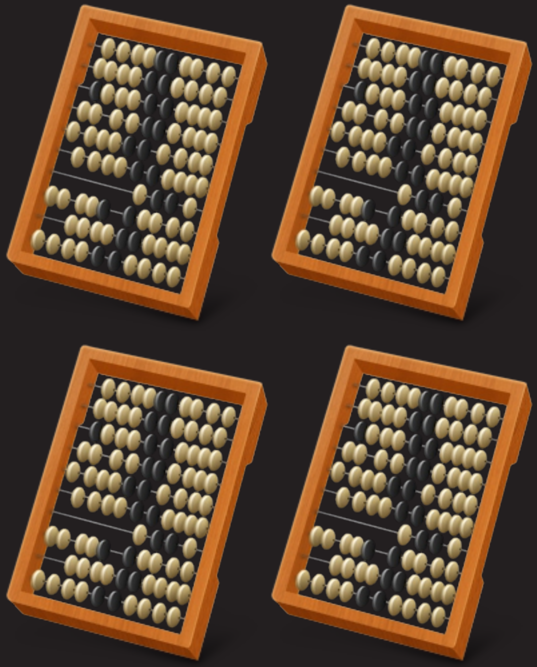
When the ensemble size, N, is increased by adding random samples, the analysis computed from this equation will converge towards the exact solution of Eq. (9) with $\boldsymbol{P}_\mathrm{e}$ and $\boldsymbol{R}_\mathrm{e}$ replaced by the exact covariance matrices $\boldsymbol{P}$ and $\boldsymbol{R}$.

Evensen [1994, 2003, 2004]

Fun with models and data:

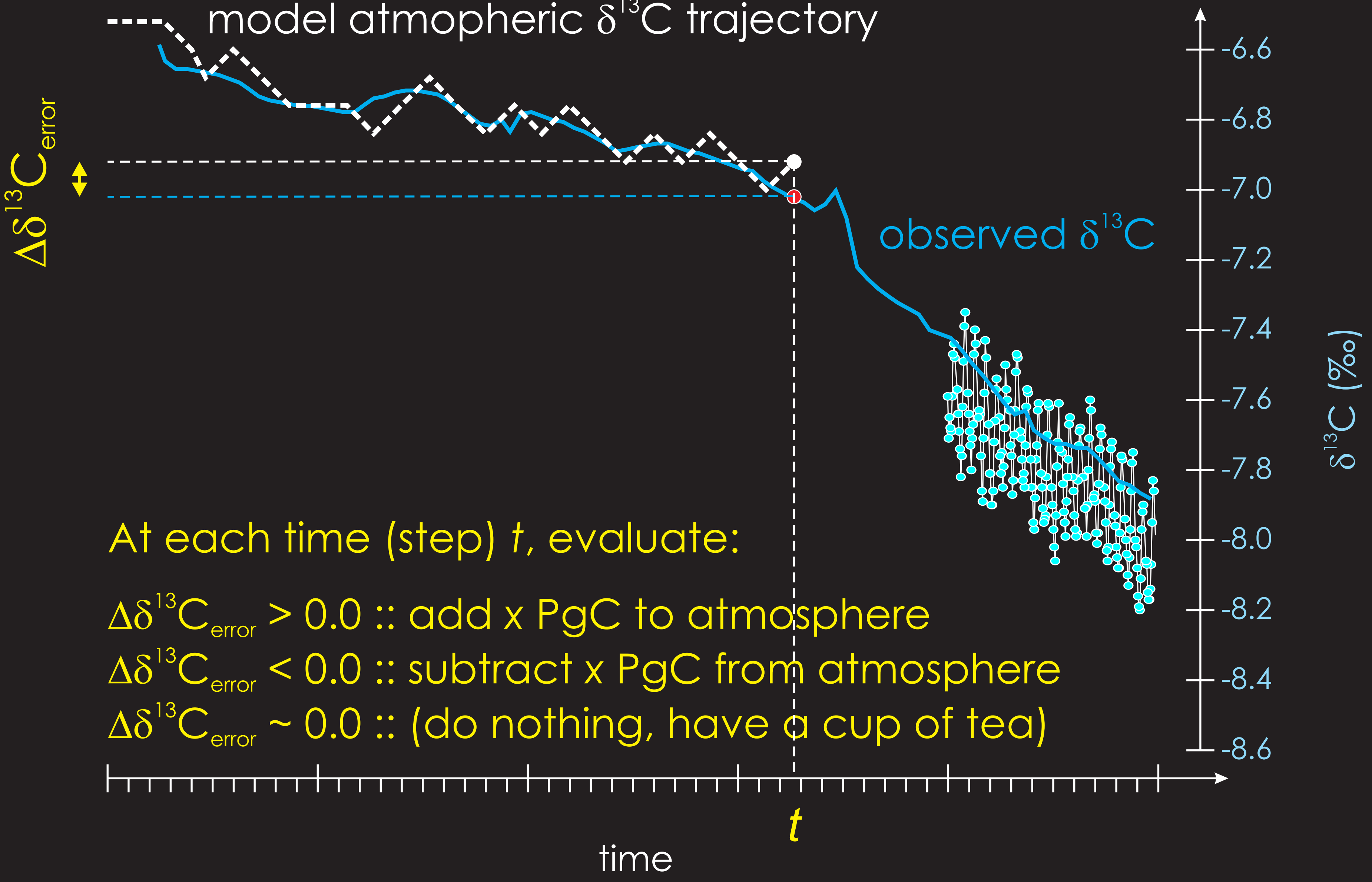
Data assimilation

CORO-MULTI
('DeConto' chipset)



Fun with models and data:

Quantifying carbon release – numerical ‘inversion’

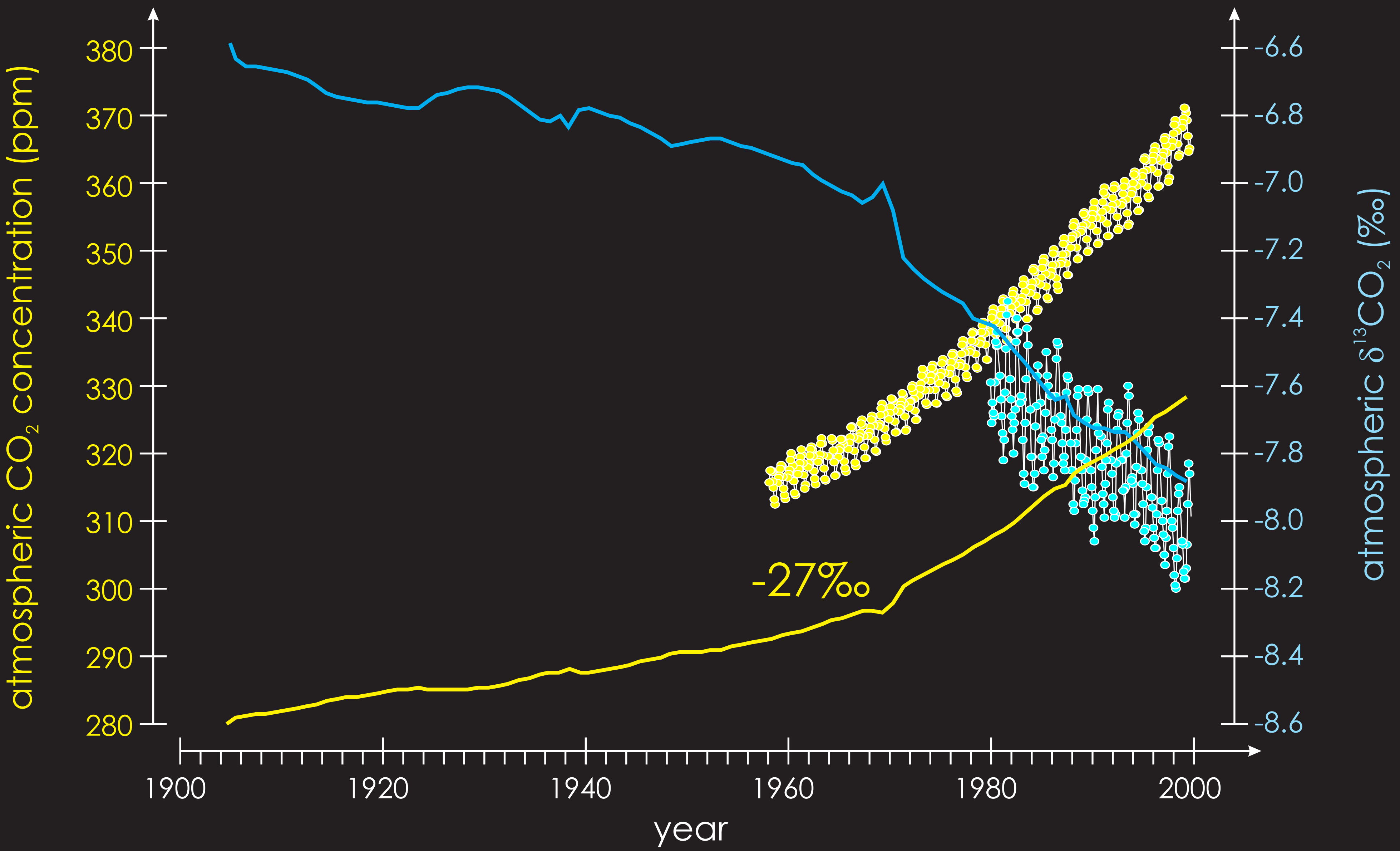


At each time (step) *t*, evaluate:

$\Delta\delta^{13}C_{error} > 0.0$ :: add x PgC to atmosphere
$\Delta\delta^{13}C_{error} < 0.0$ :: subtract x PgC from atmosphere
$\Delta\delta^{13}C_{error} \sim 0.0$ :: (do nothing, have a cup of tea)

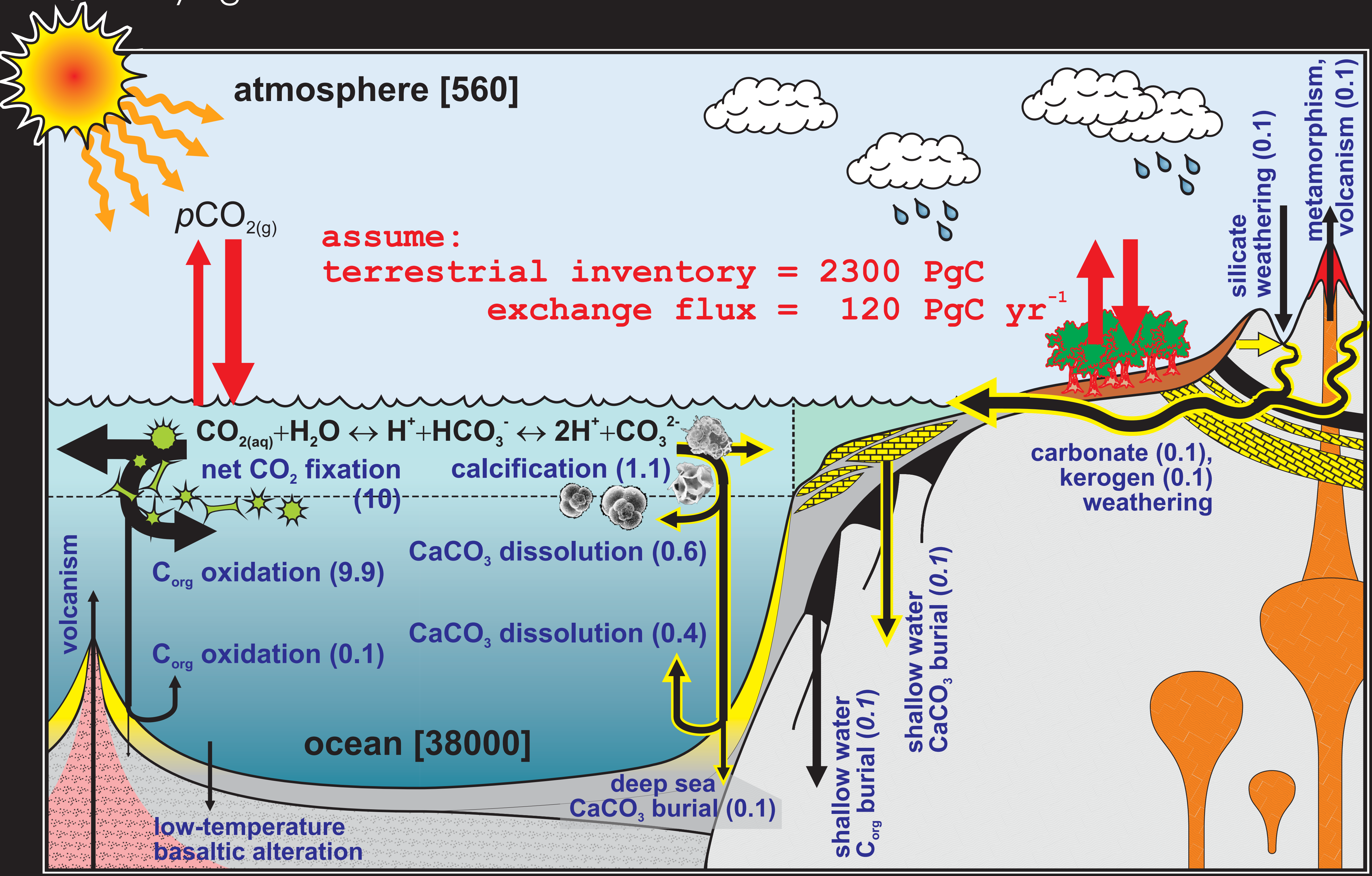
Fun with models and data:

Quantifying carbon release – numerical ‘inversion’



Fun with models and data:

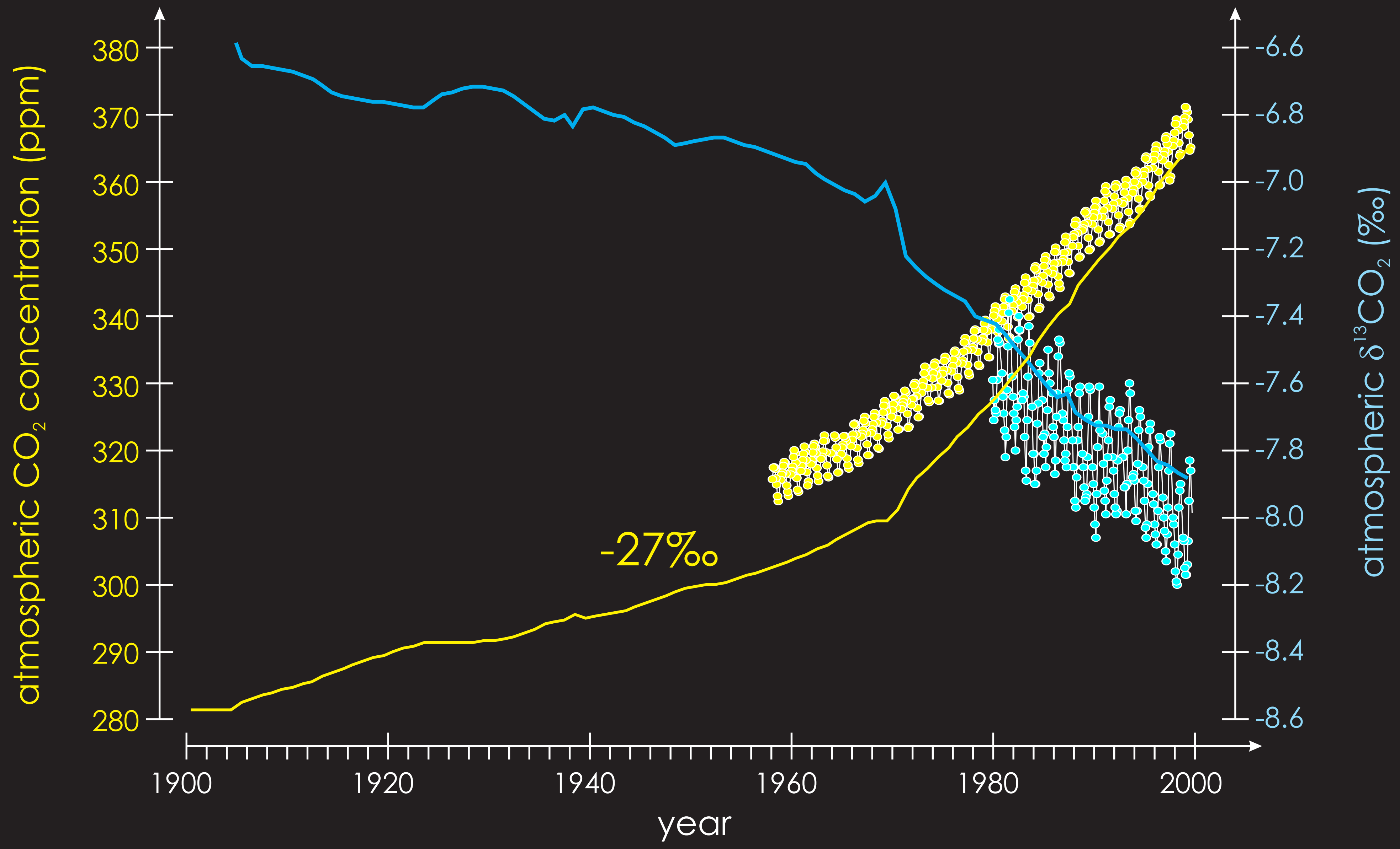
Quantifying carbon release – numerical ‘inversion’



assume:
terrestrial inventory = 2300 PgC
exchange flux = 120 $PgC\ yr^{-1}$

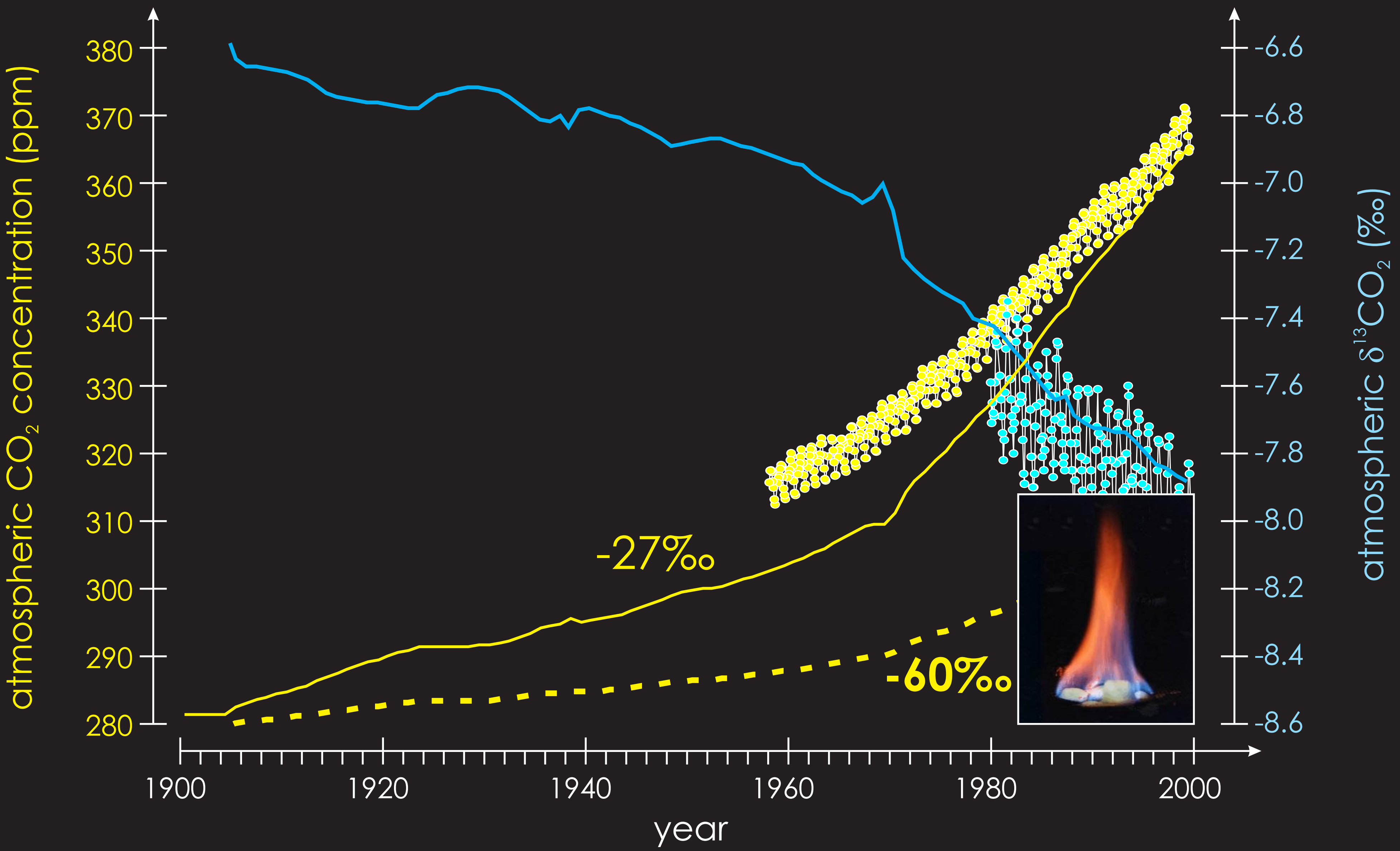
Fun with models and data:

Quantifying carbon release – numerical ‘inversion’



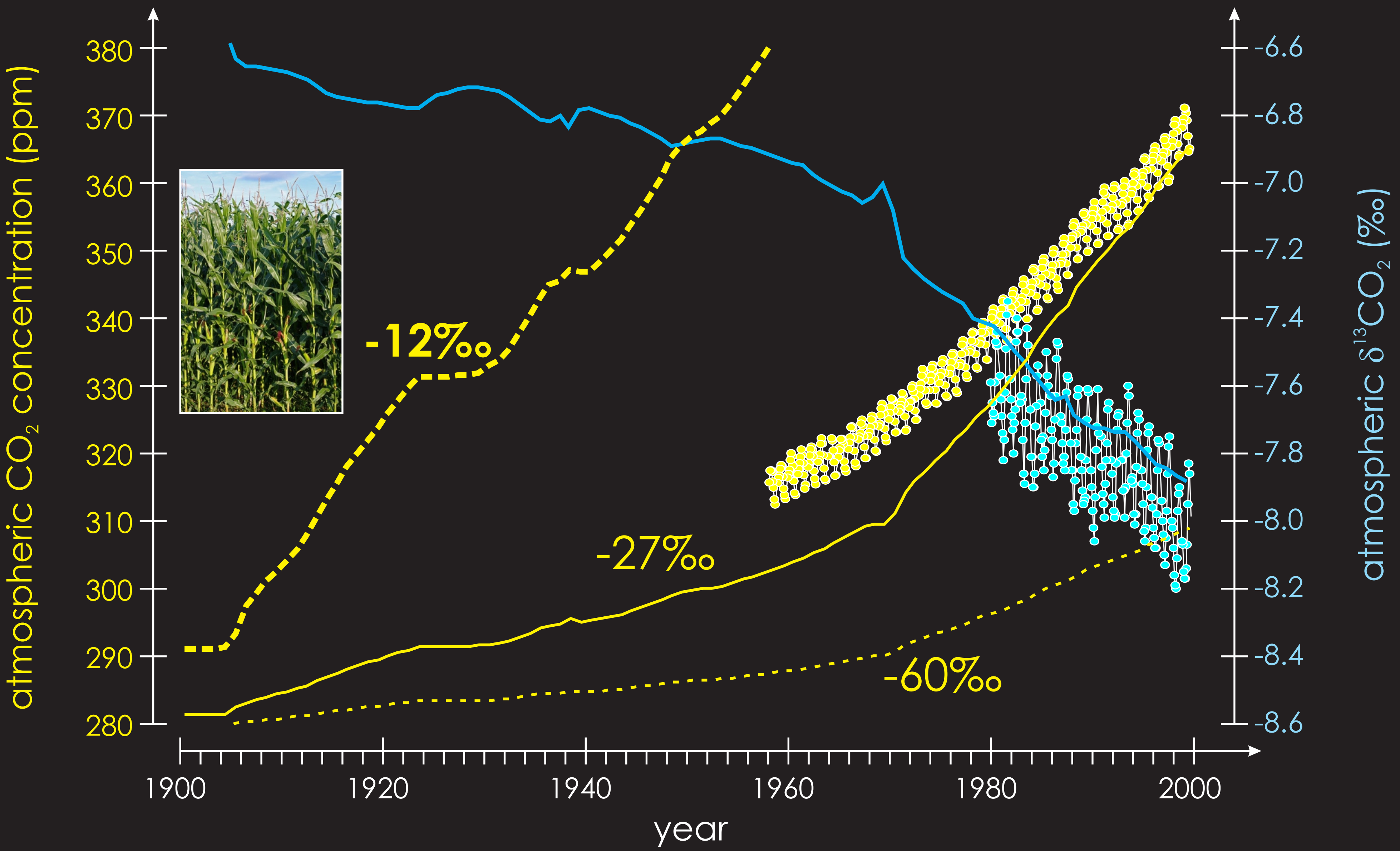
Fun with models and data:

Quantifying carbon release – numerical 'inversion'

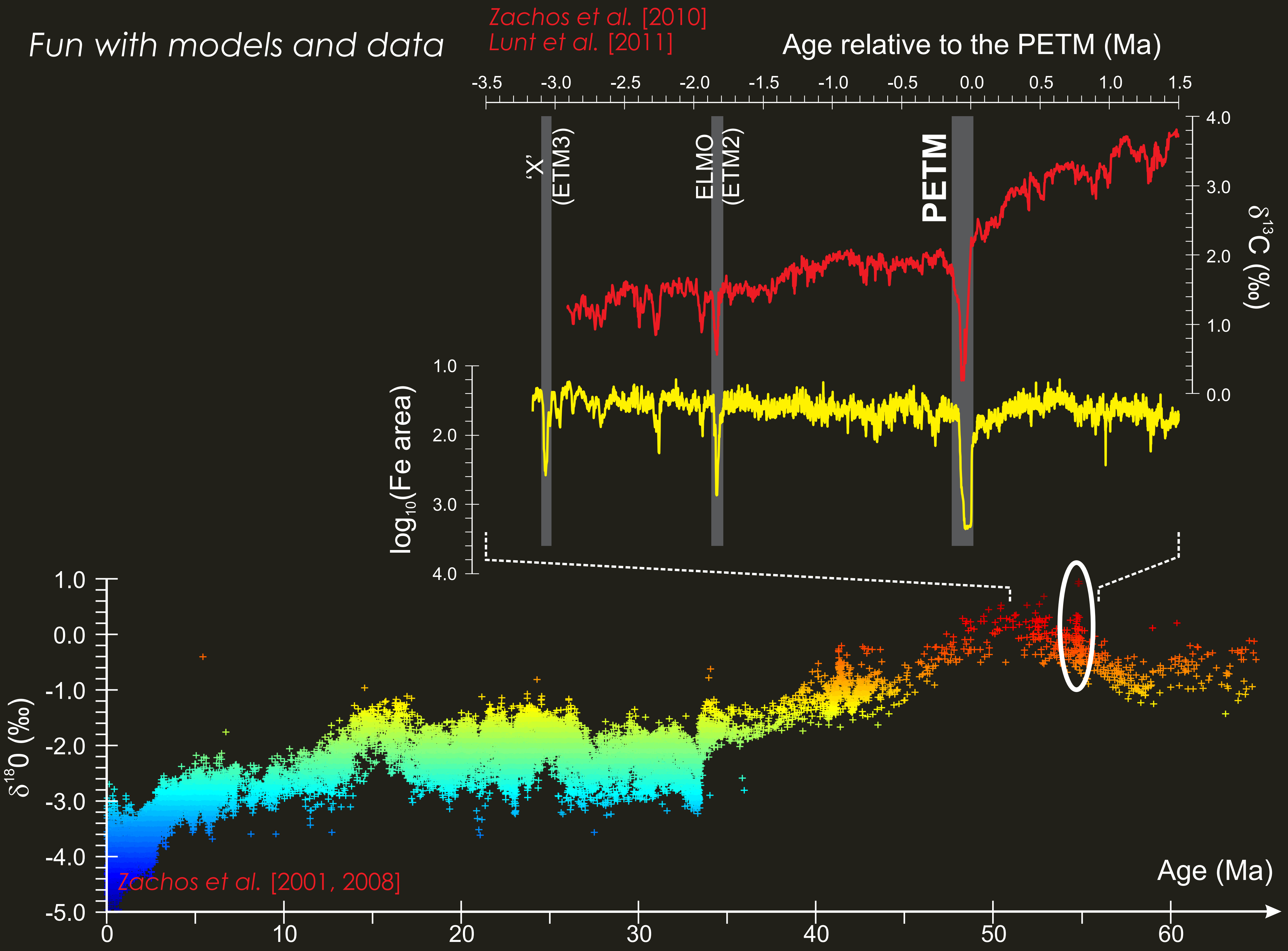


Fun with models and data:

Quantifying carbon release – numerical ‘inversion’



Fun with models and data

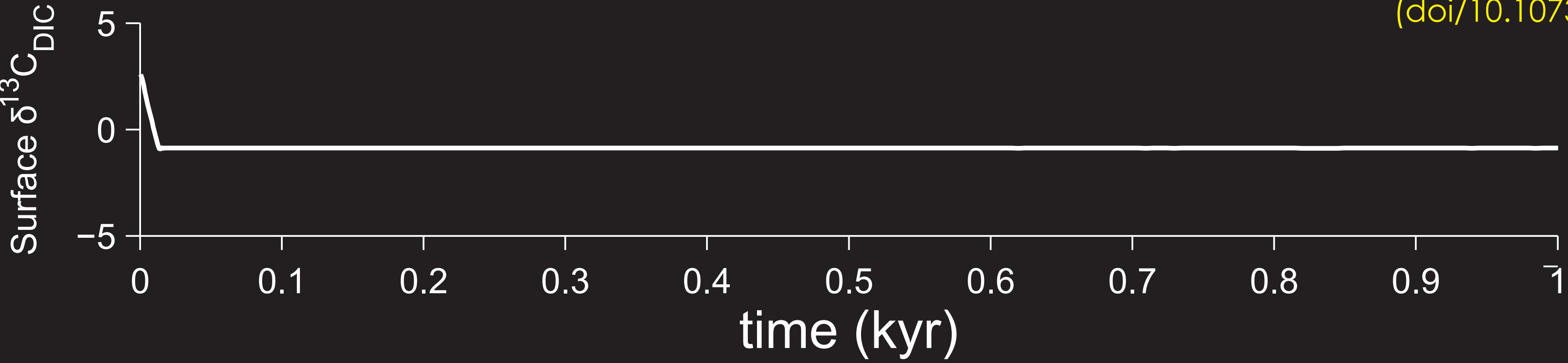


Zachos et al. [2010]
Lunt et al. [2011]

Zachos et al. [2001, 2008]

Fun with models and data: Surface δ^{13}C record inversions

Wright and Schaller [2013]
(doi/10.1073/pnas.1309188110)



Fun with models and data: Surface $\delta^{13}C$ record inversions

Wright and Schaller [2013]
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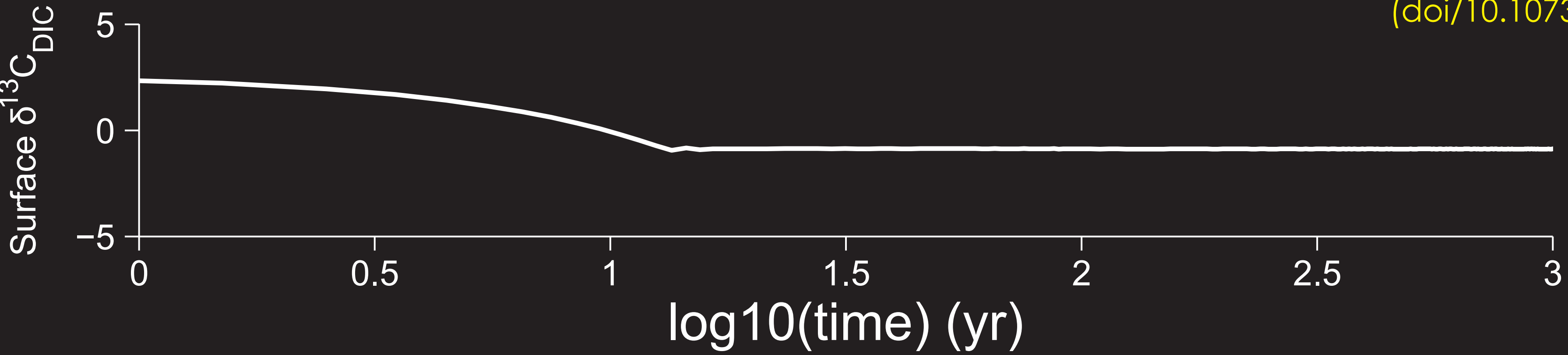
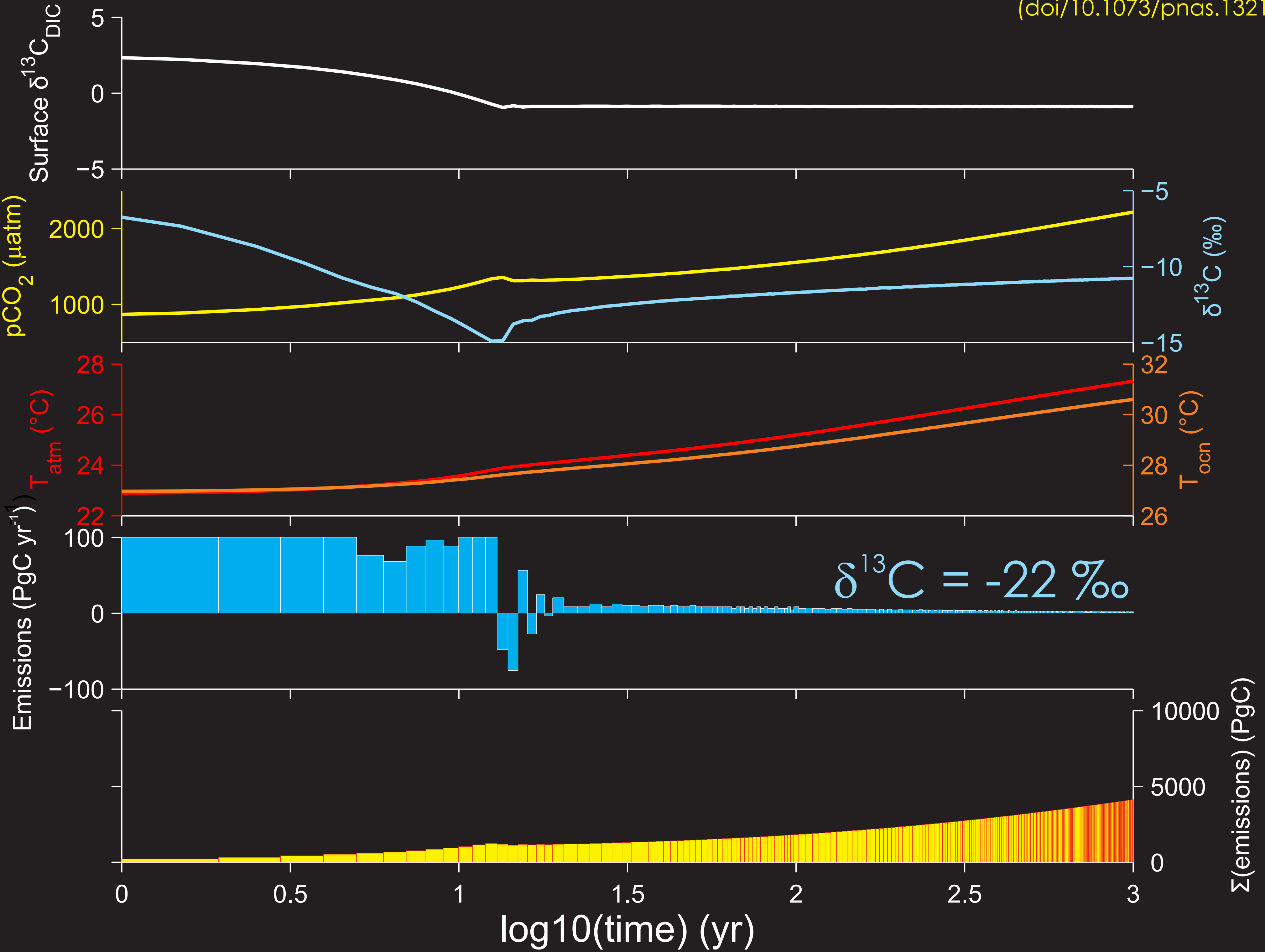


Fun with models and data: Surface δ^{13}C record inversions

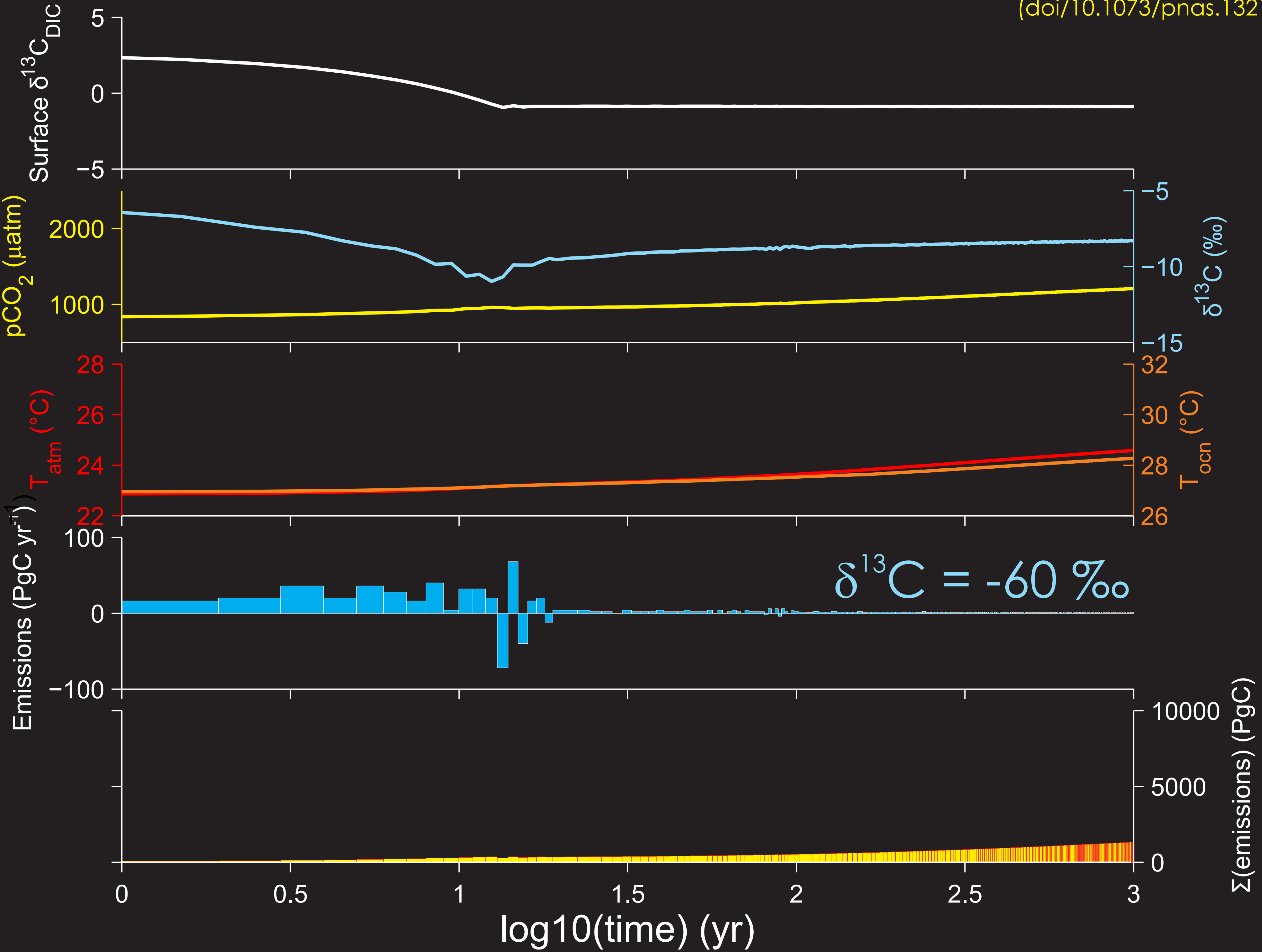
Zeebe et al. [2014]
(doi/10.1073/pnas.1321177111)



Fun with models and data: Surface δ^{13}C record inversions

Zeebe et al. [2014]

(doi/10.1073/pnas.1321177111)



Fun with models and data: Surface δ^{13}C record inversions

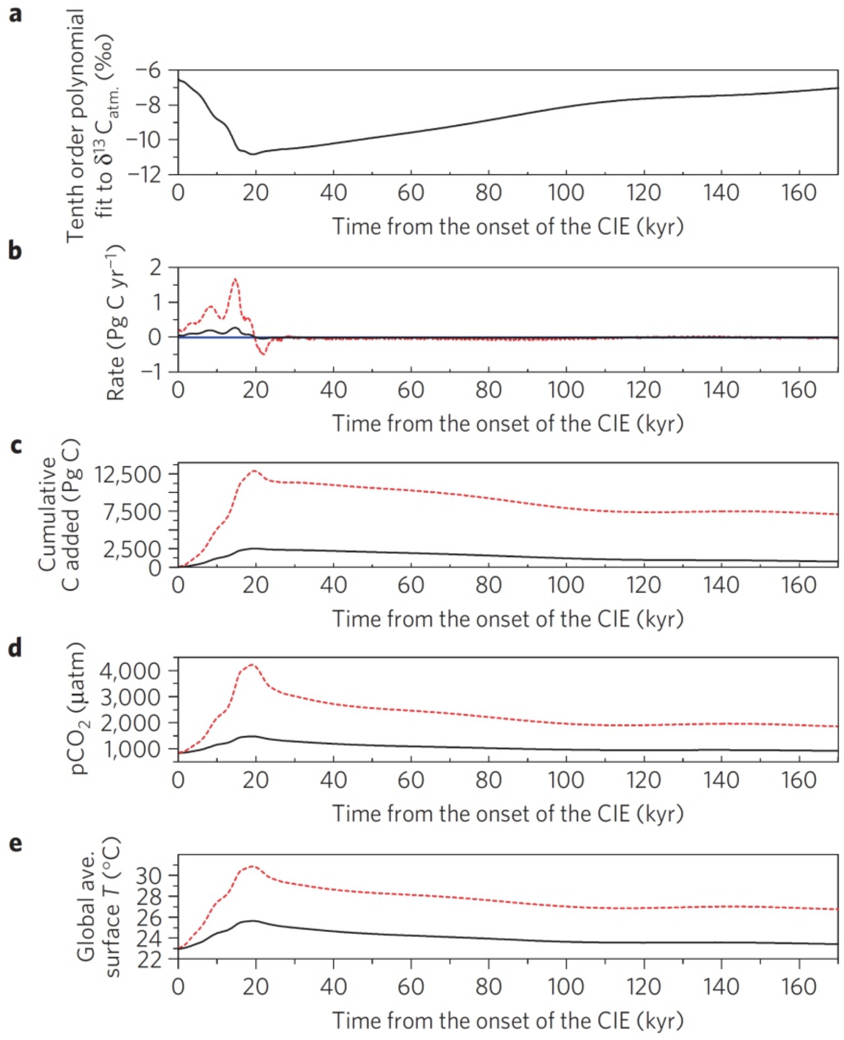
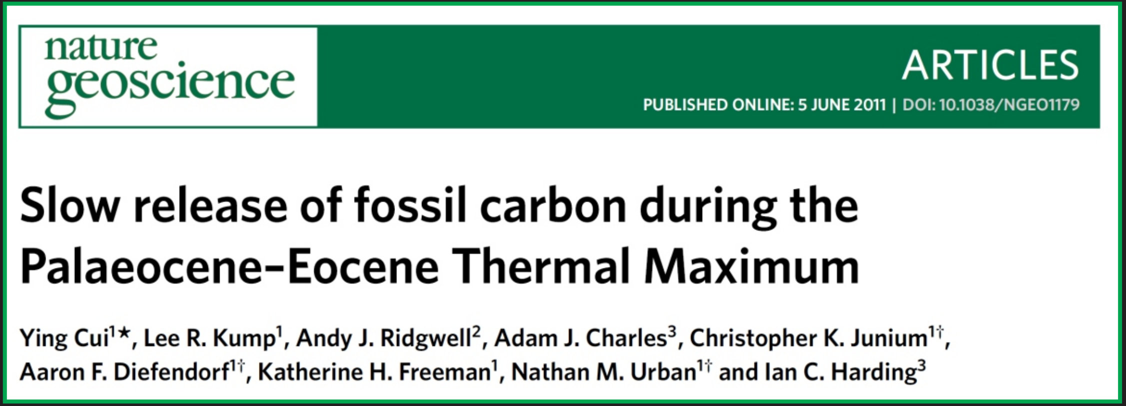


Figure 4 | Model results of the PETM carbon release rate and cumulative amount of carbon added versus time from the onset of the CIE (535 mbs) (age model is from ref. 2). a, $\delta^{13}C_{atm}$ that we used to force GENIE. **b**, Model results of the PETM carbon release rate. **c**, Model results of the cumulative amount of carbon added. **d**, Model results of the PETM atmospheric pCO_2. **e**, Model results of the PETM global average temperature (°C). The two best-fit simulations are shown in **b–e**:(1) CH_4 simulation (black solid line); (2) C_{org} simulation (red dotted line). Both simulations are with bioturbation on.



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ARTICLES

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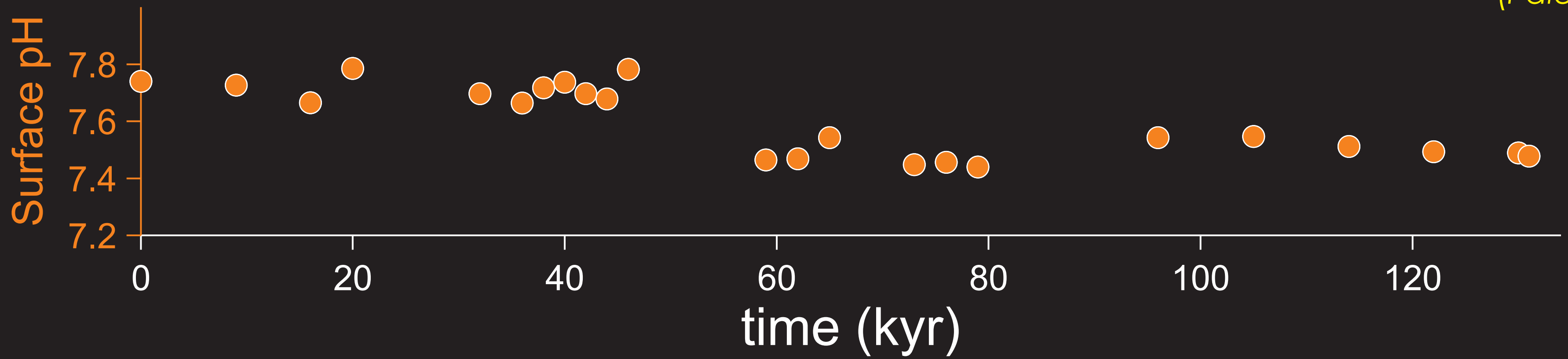
Slow release of fossil carbon during the Palaeocene–Eocene Thermal Maximum

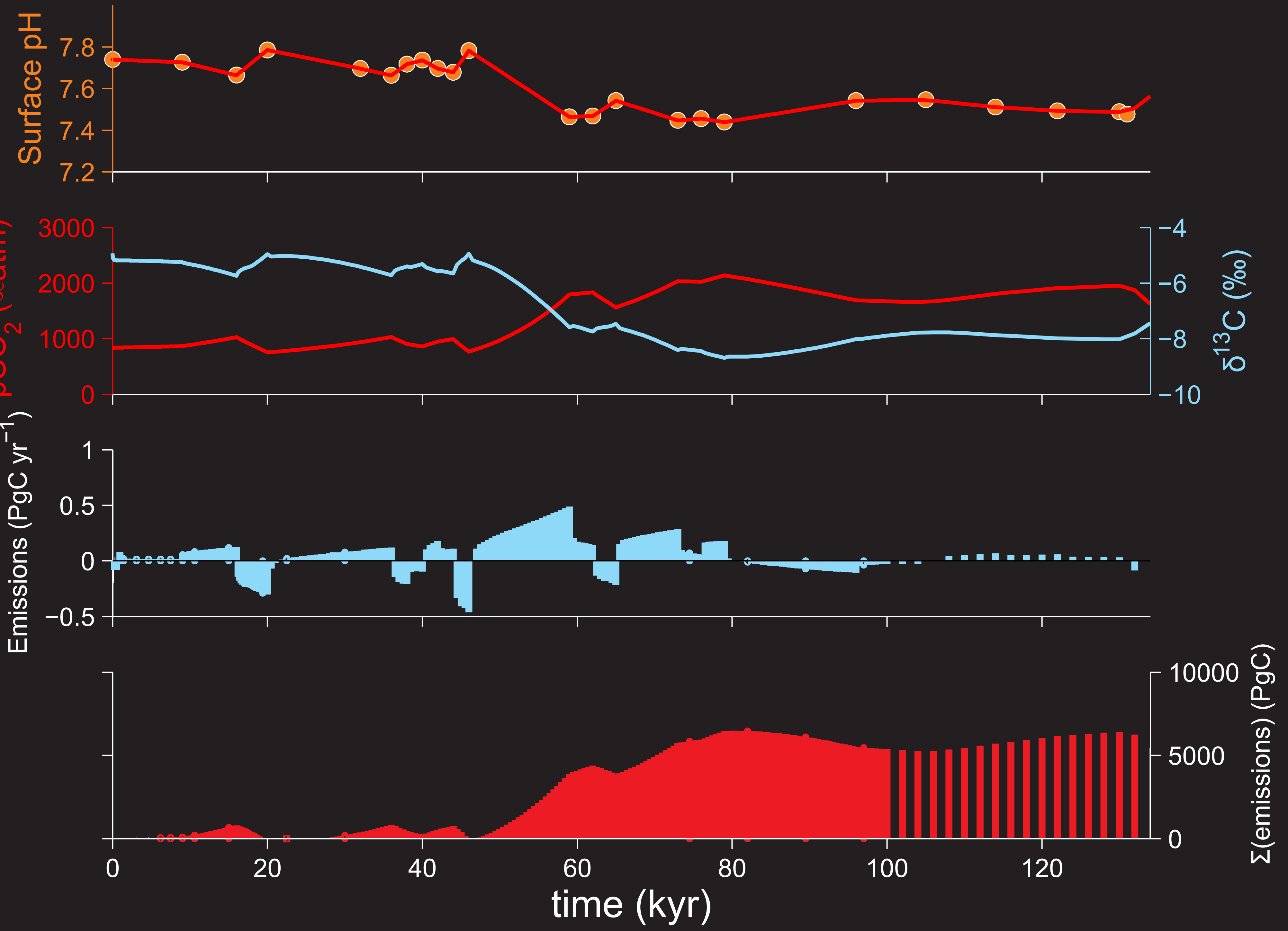
Ying Cui[1]*, Lee R. Kump[1], Andy J. Ridgwell[2], Adam J. Charles[3], Christopher K. Junium[1†], Aaron F. Diefendorf[1†], Katherine H. Freeman[1], Nathan M. Urban[1†] and Ian C. Harding[3]

Fun with models and data: Surface $\delta^{13}C$ record inversions

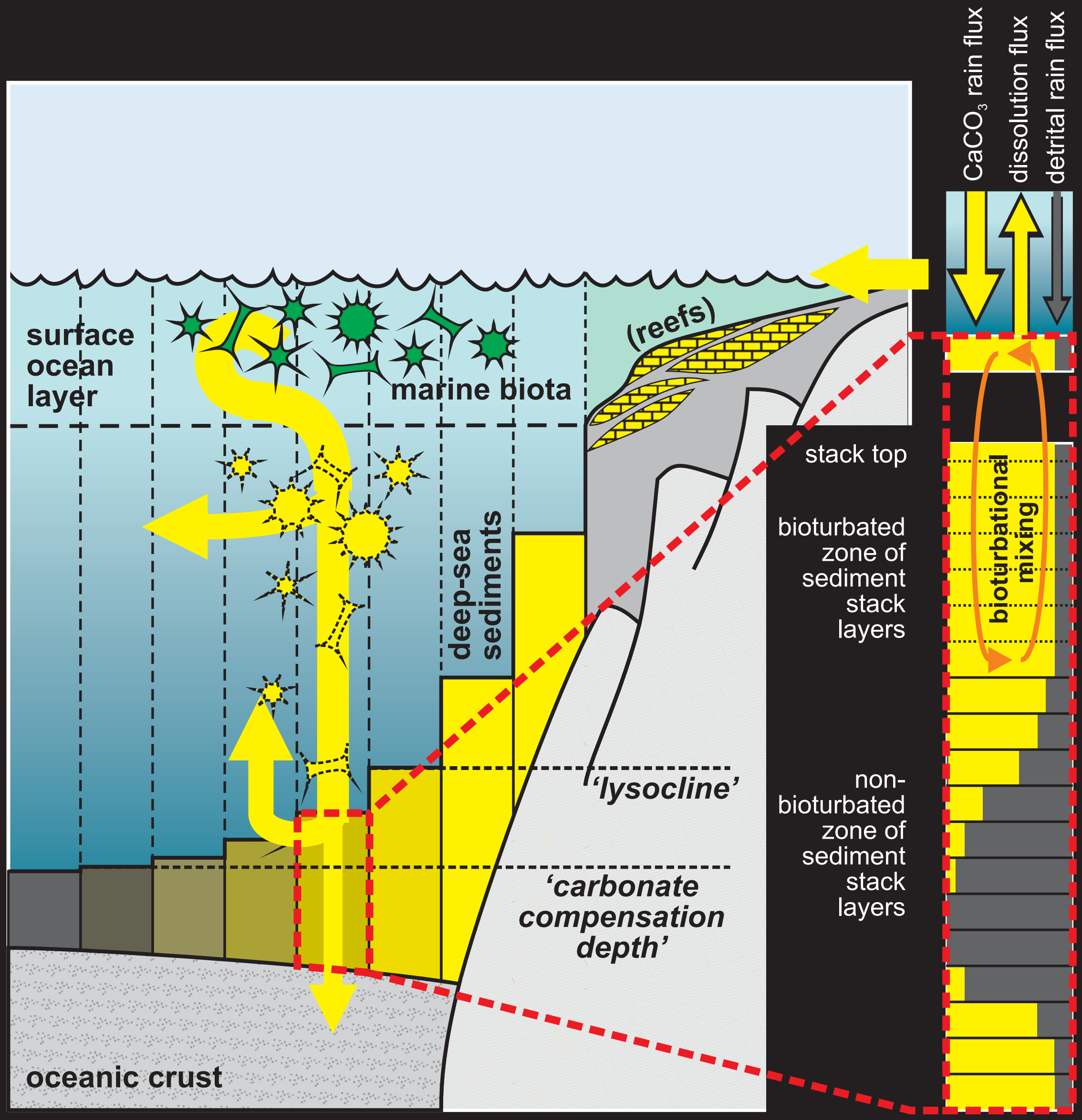
Penman et al. [2014]
(Paleoceanography)



Fun with models and data: Surface δ^{13}C record inversions

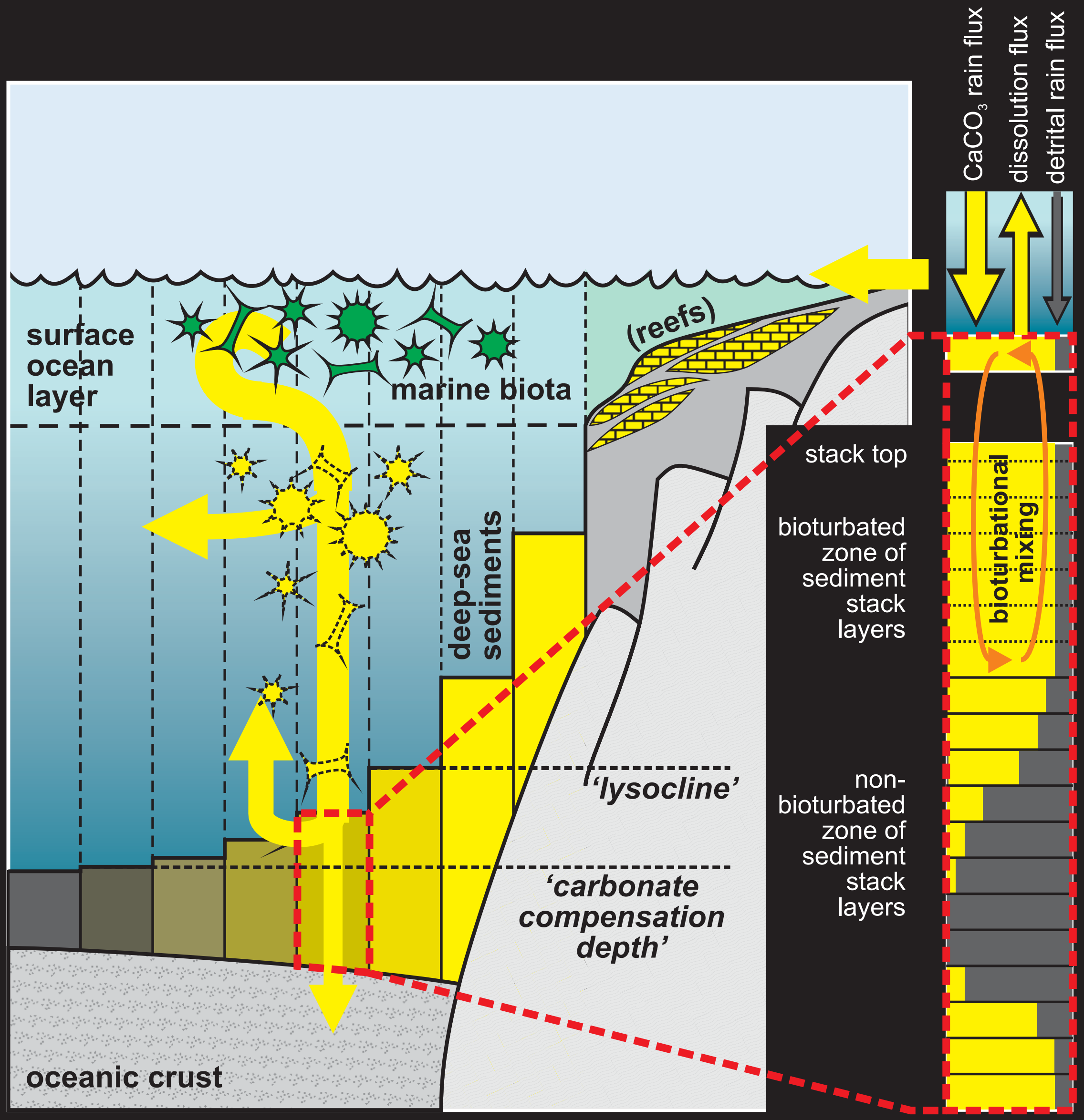


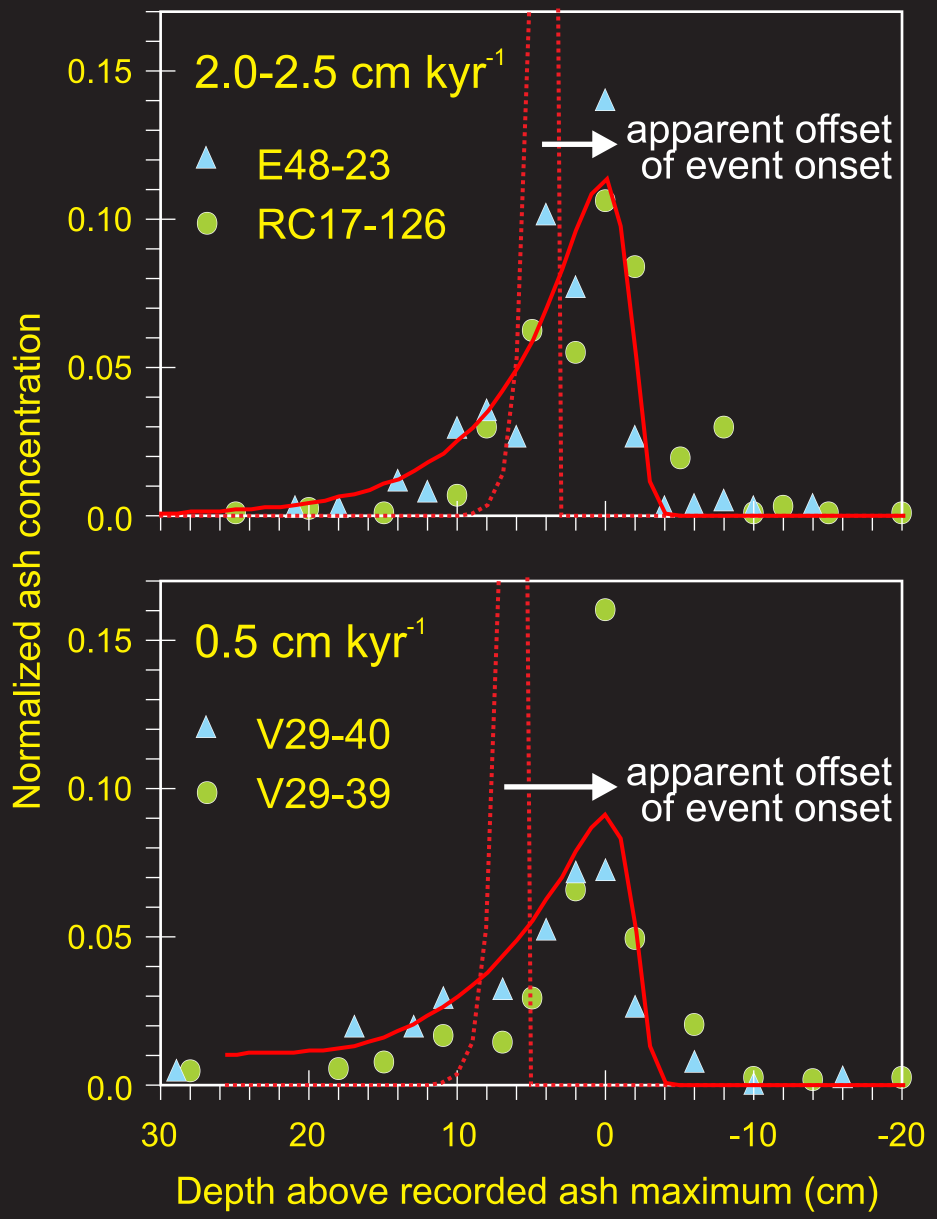
Fun with models and data: Benthic δ^{13}C record inversions



Problem #1: 'target' (proxy record) is remote (in time) from the carbon input (assuming to the atmosphere).

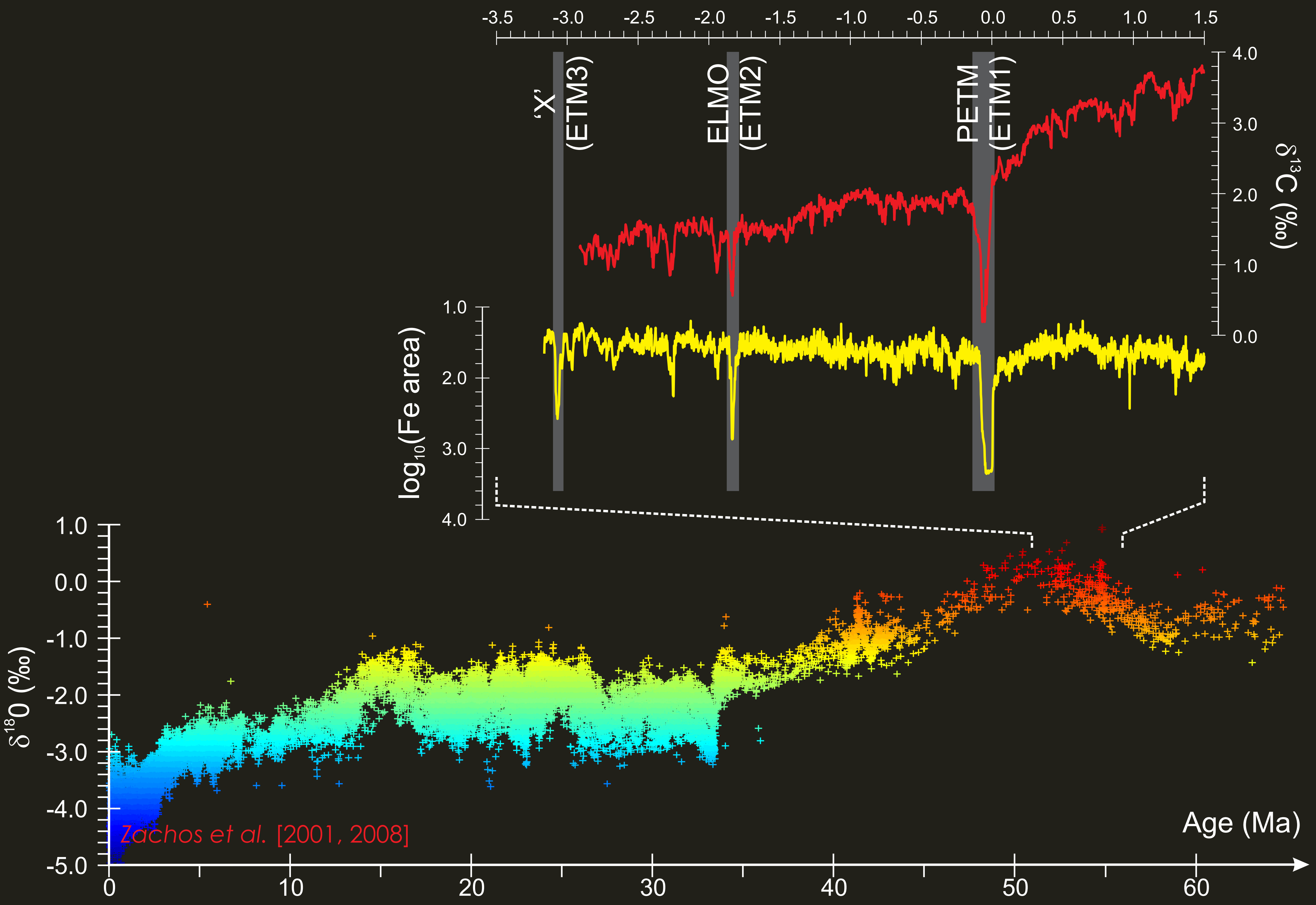
Fun with models and data



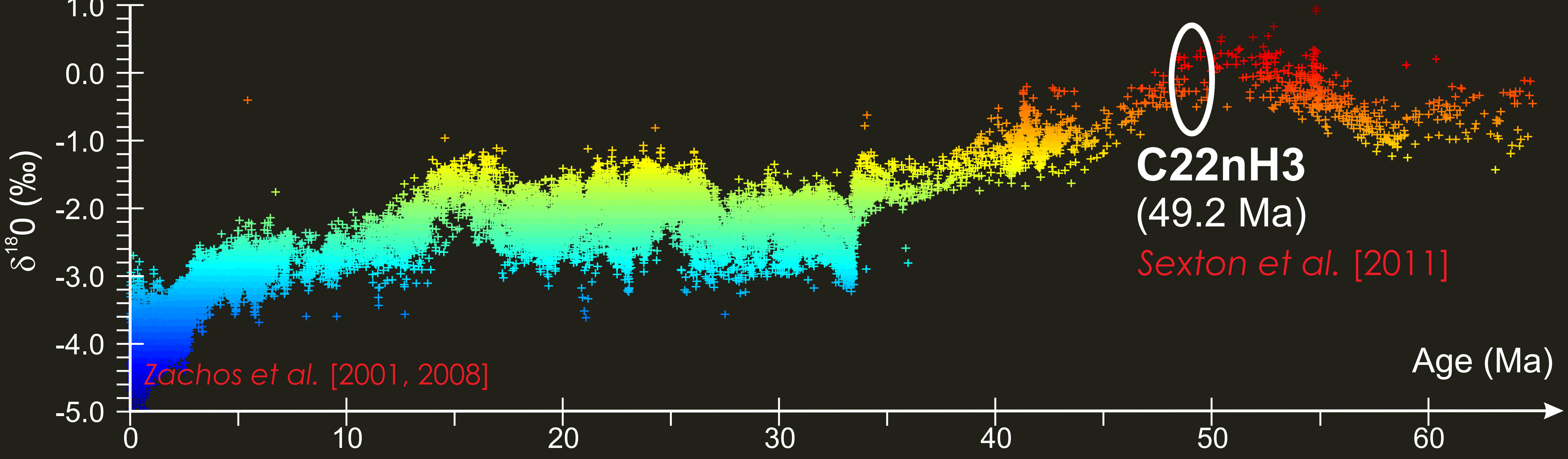


Problem #1: sediments are mixed (bioturbated).

Fun with models and data

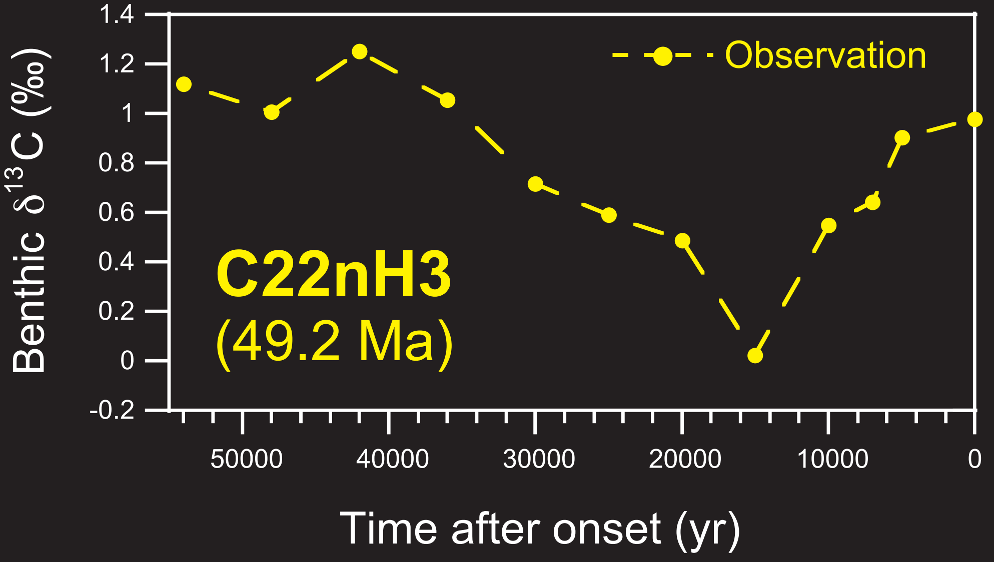


Fun with models and data



Fun with models and data:
Inverting benthic records

Kirtland-Turner and Ridgwell [2013]



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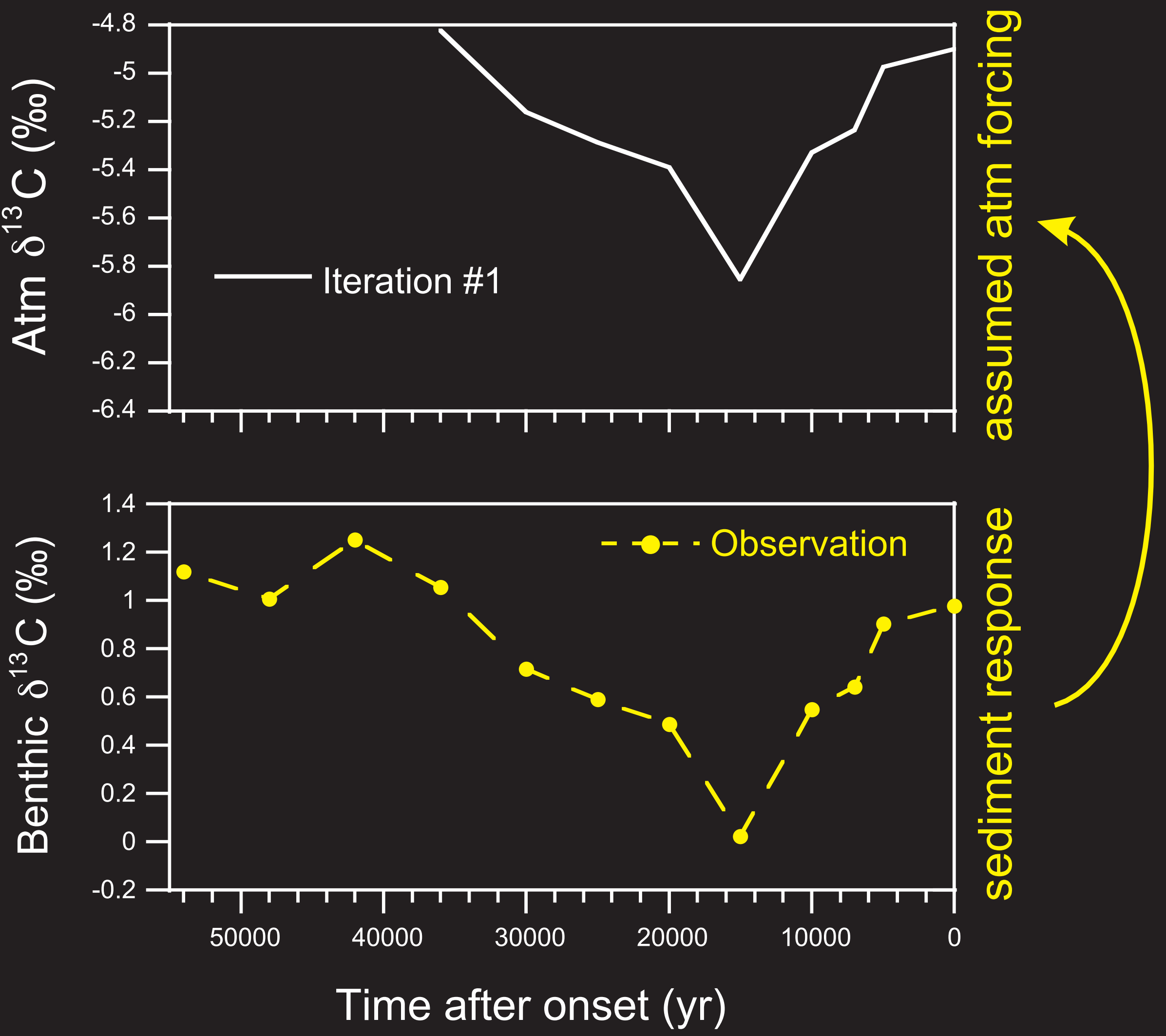
Inverting benthic records

Kirtland-Turner and Ridgwell [2013]



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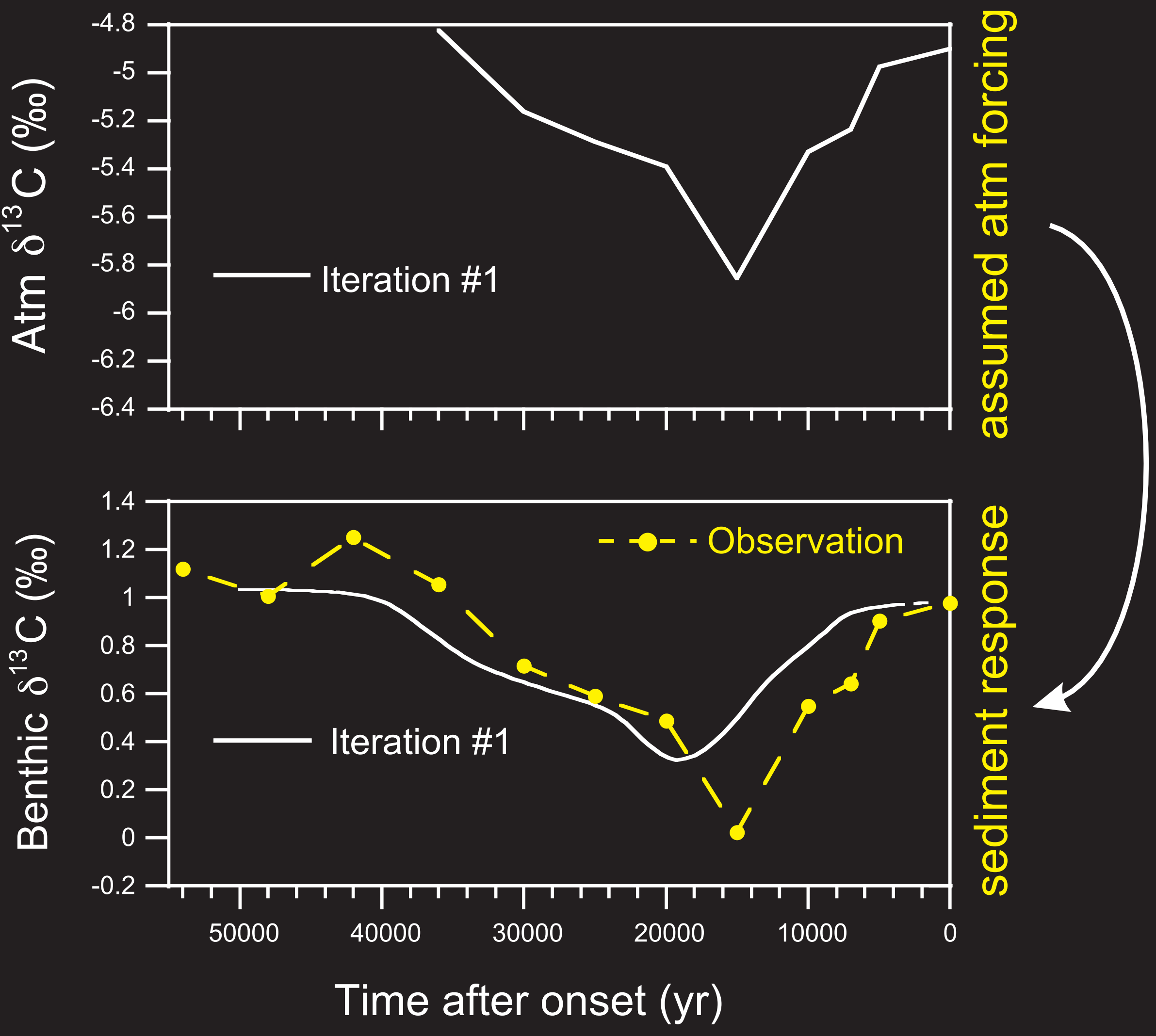

Fun with models and data:
Inverting benthic records

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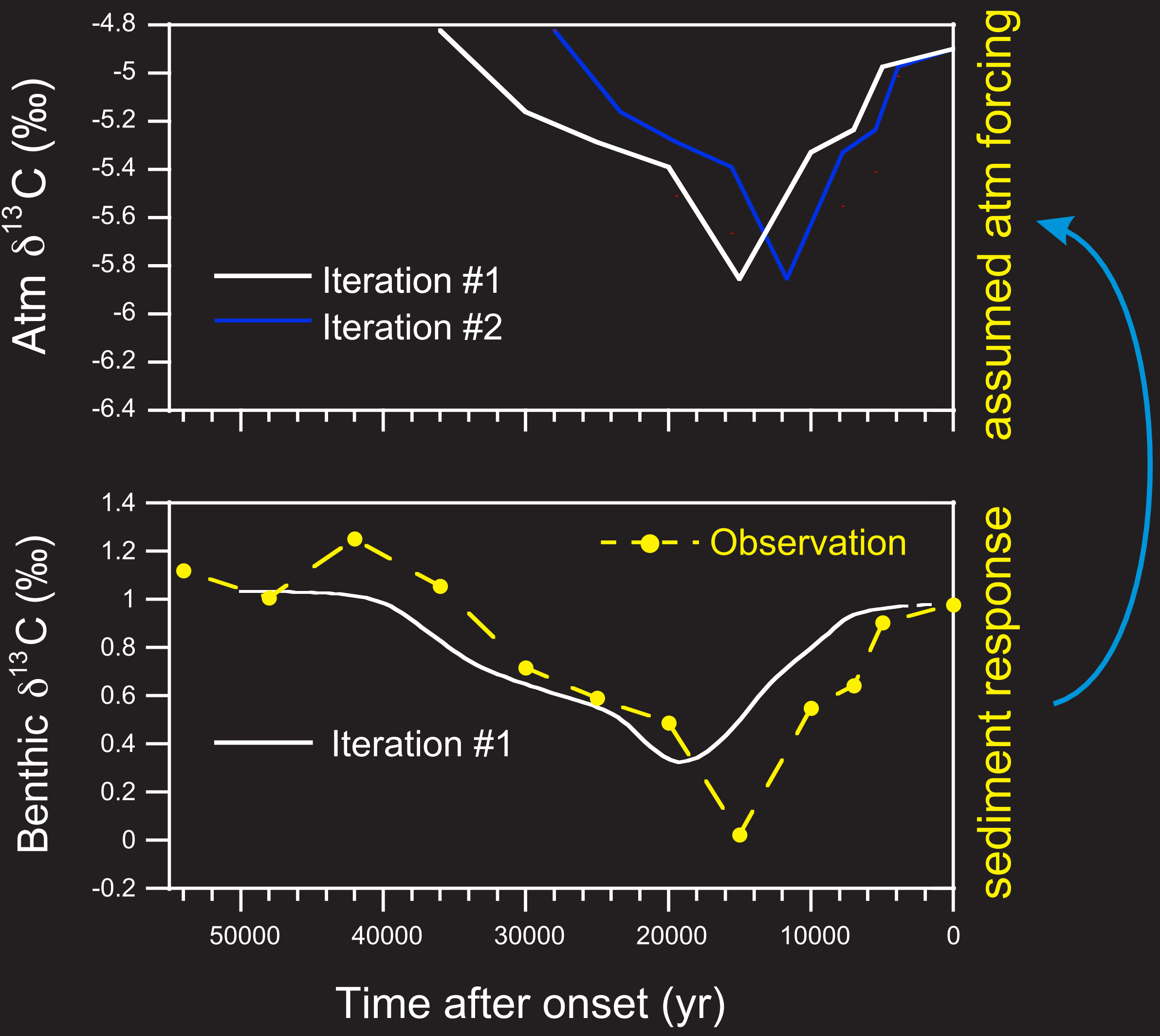

Fun with models and data: Inverting benthic records

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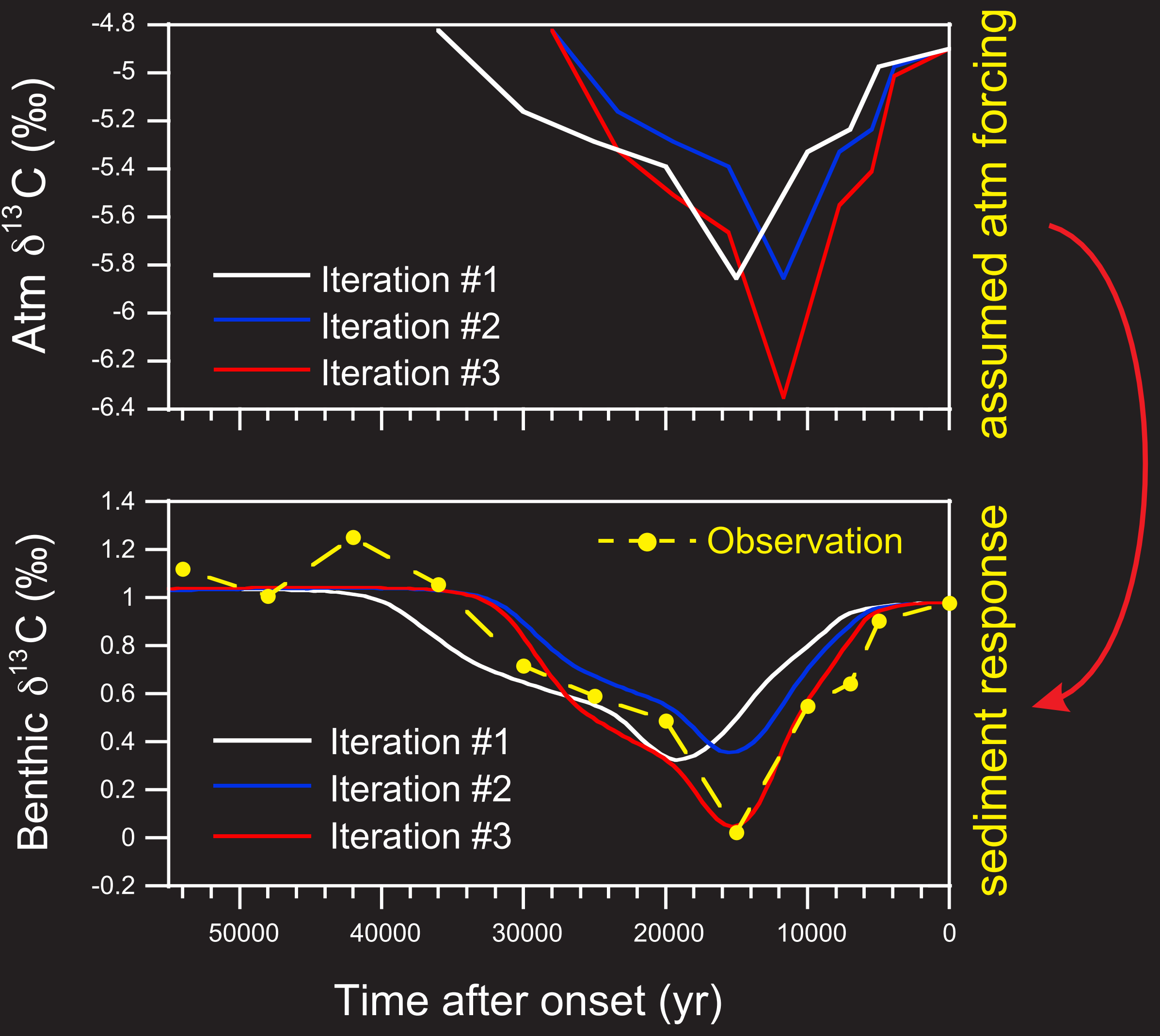

Fun with models and data: Inverting benthic records

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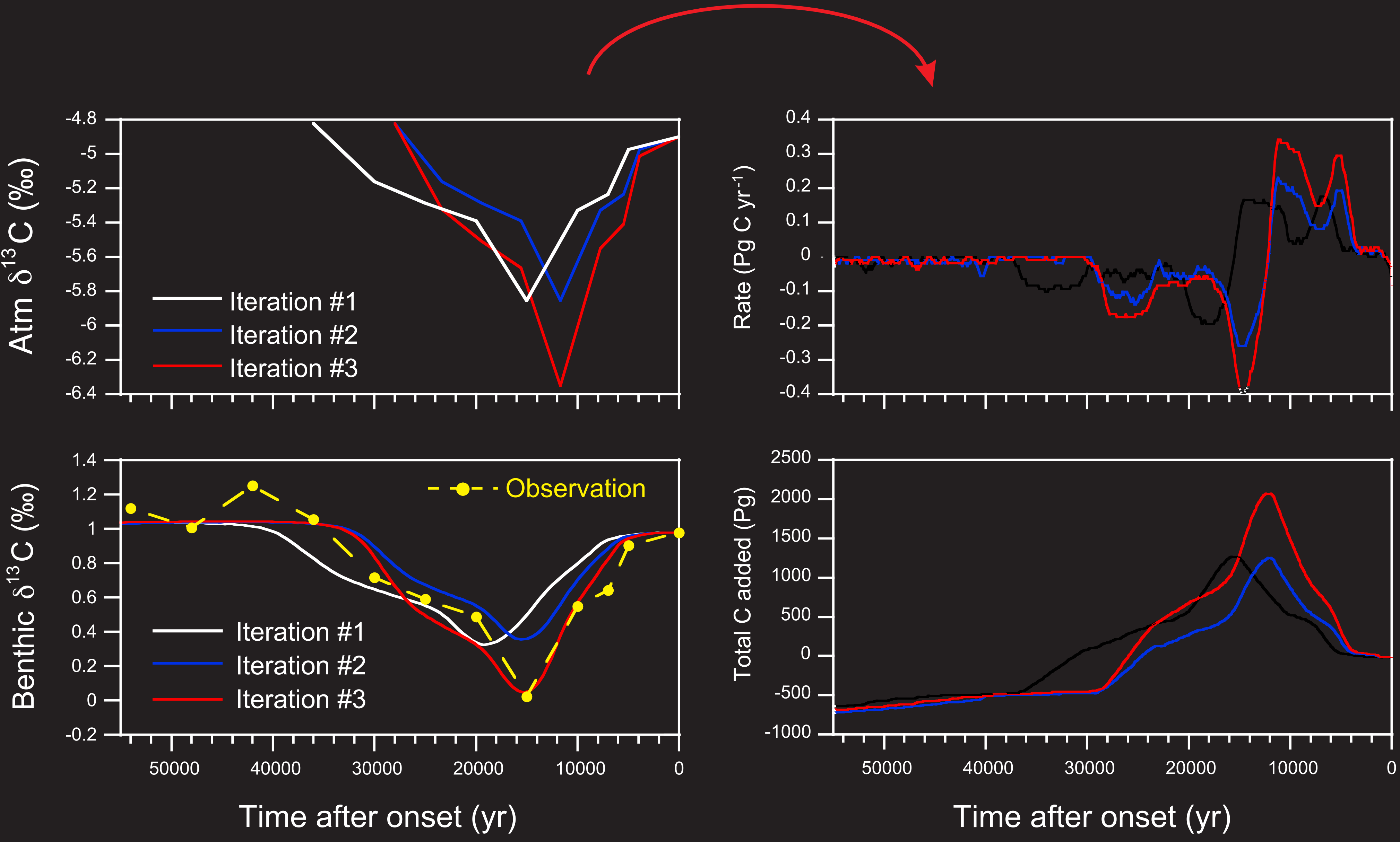
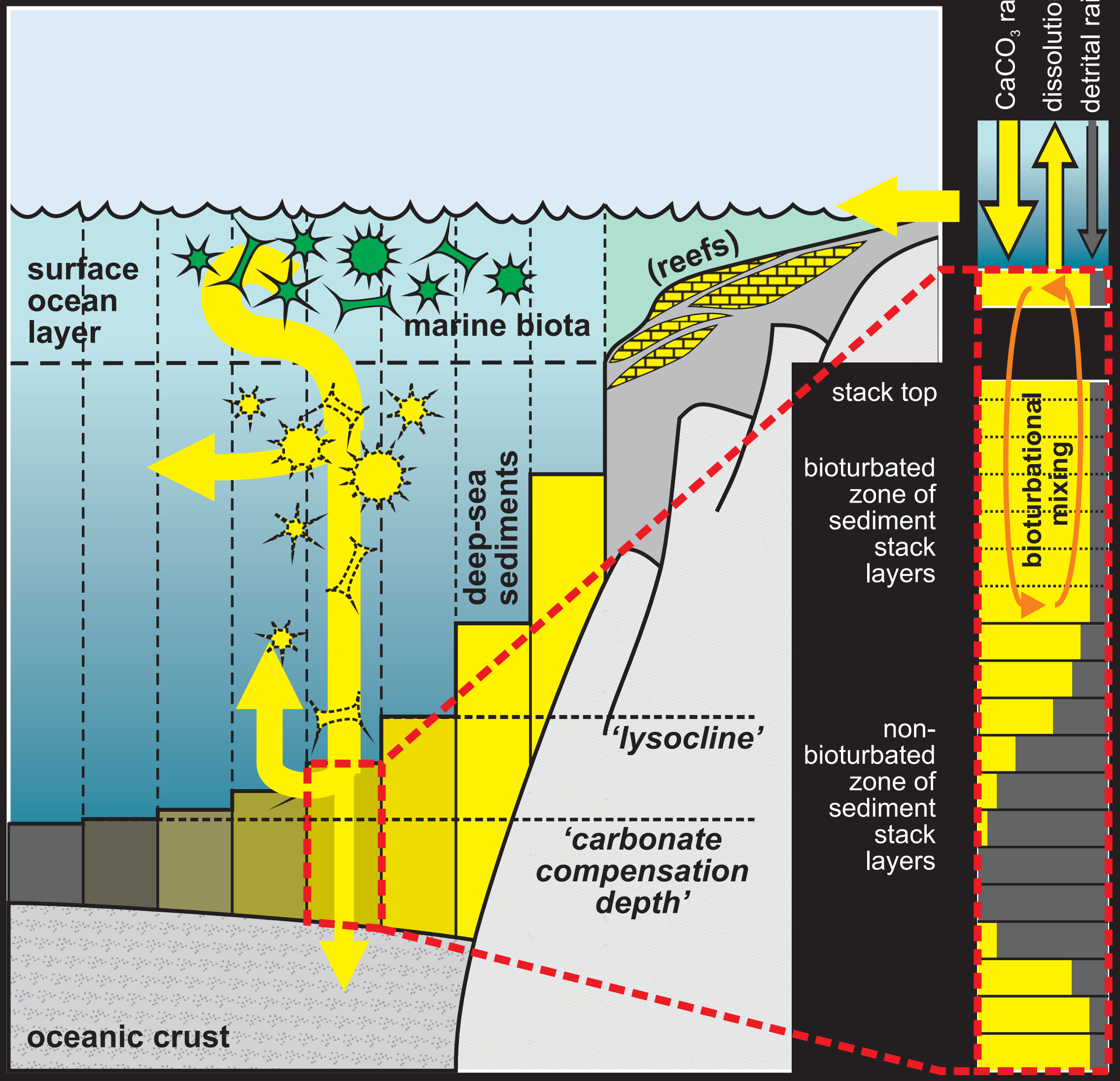
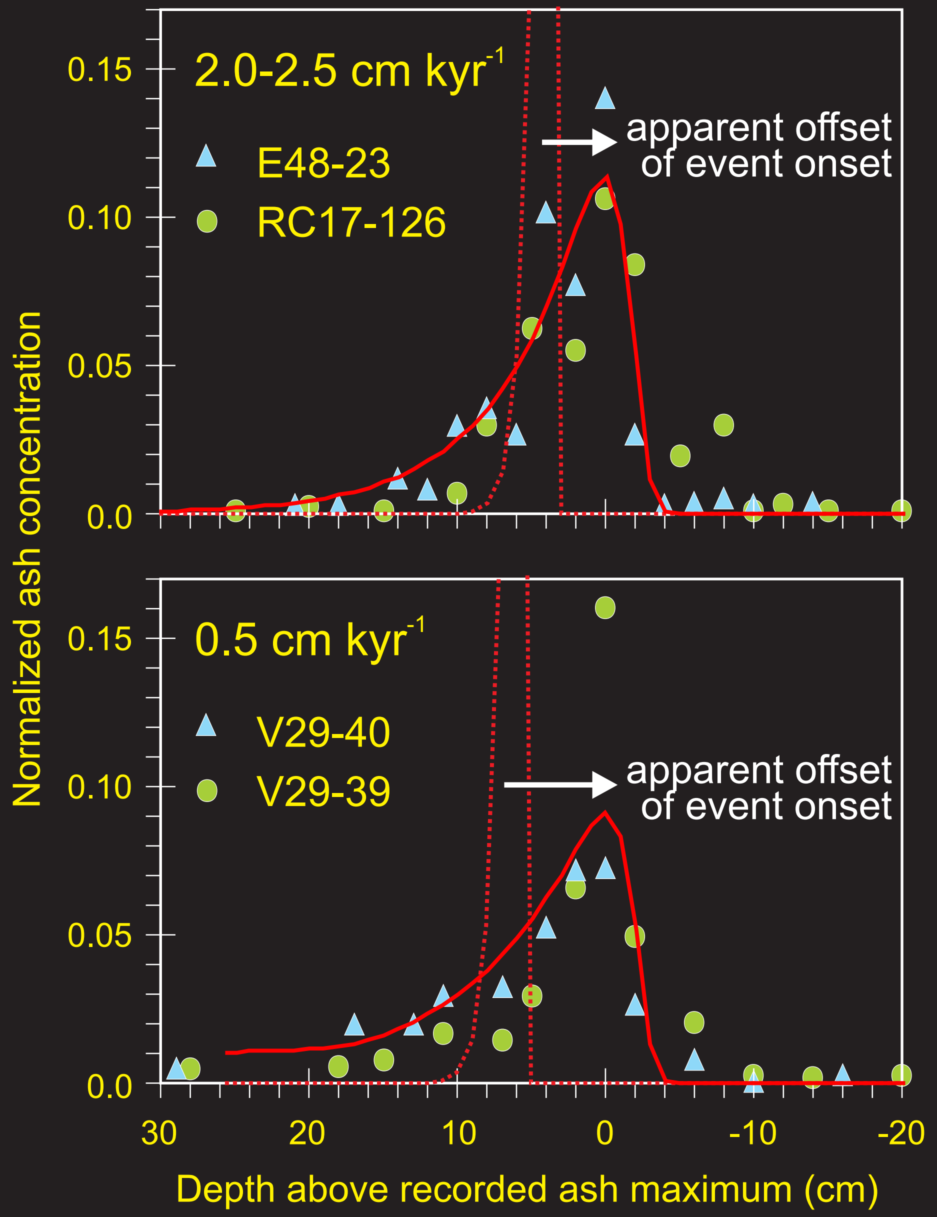


Fun with models and data





The End

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Sandy Kirtland-Trurner, Suzi Jennions, Ellen Thomas, Dani Schmidt

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